Bayesian Network Modeling of Acoustic Sensor Measurements

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***Abstract*— Control and optimization of acoustic sensors can significantly impact the effectiveness of sonar deployment in variable and uncertain underwater environments. On the other hand, the design of optimal control systems requires tractable models of system dynamics, which in this case include acoustic-wave propagation phenomena. High-fidelity acoustic models that capture the influence of environmental conditions on wave propagation involve partial differential equations (PDEs), and are computationally intensive. Also, by relying on the numerical solution of PDEs for given boundary and initial conditions, they do not provide closed-form functional forms for the propagation loss or other output variables. In this paper, a simple Bayesian network (BN) model of acoustic propagation is presented for use in sonar control. The performance of the BN model compares favorably to that of a radial basis function neural network. Additionally, the sensor range dependency on spatial and temporal coordinates can be estimated and utilized to compute optimal sonar control strategies.**

I. Introduction

On-going developments in acoustic sensor technologies and signal processing are producing sonar systems with capabilities for automatic reconfiguration and deployment. The tactical objectives of these systems, such as, target detection, classification and tracking, can benefit significantly from automatic sonar control and optimization techniques that account for the environmental influence on the acoustic measurements performance. A typical configuration for the control of a sonar system, such as, a towed array or a sonobuoy network [1,2], is shown in Fig. 1. The control system requires the use of acoustic and environmental models to quantify the effects that the ocean environment has on the propagation of acoustic waves.

Acoustic models play a critical role in the control process and must be chosen carefully depending on the control objectives, because their applicability is limited by what environmental conditions are observable. A comprehensive review of the literature on acoustic propagation models is provided in [1]. These models include Parabolic Equation Models, Ray Models, Normal Mode Models and Fast Field Models. The parabolic equation approach replaces the elliptic-reduced wave equation with a parabolic equation (PE) and is the most accurate approach for modeling range-dependent 3D ocean acoustic propagation [4-7]. Range-dependent Acoustic Model (RAM) is a FORTRON code based on the split-step Padé approximation solutions [6,7]. Recently developed techniques in PE modeling are very effective for solving range-dependent acoustic propagation loss problems for passive sonar systems. However, they cannot be used for designing optimal control and decision strategies because they rely on numerical integration.

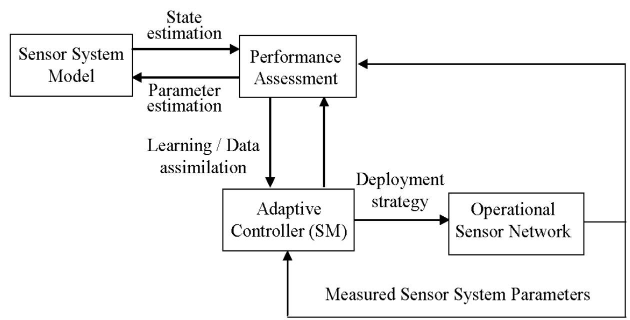

Fig. 1. A typical configuration of sonar controller

In this paper, a Bayesian network (BN) model of acoustic propagation is trained using RAM to obtain a closed-form representation of the environmental conditions influence on sonar measurements. Another advantage over existing acoustic models is that it rapidly approximates acoustic model calculation, with a reasonable tradeoff between computational complexity and model accuracy. Consequently, the sensor range dependency on spatial and temporal coordinates can be estimated and utilized to compute optimal deployment strategies for the sonar system.

II. Background

A. *Acoustic Propagation Models and RAM Software*

Acoustic propagation models are based on a three-dimensional, time-dependent wave equation [1]. In cylindrical coordinates, a simplified linear, hyperbolic, second-order, partial differential equation is,

This work was supported by the Office of Naval Research Young Investigator Program (Code 321).

$$\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}+\frac{\partial^2 p}{\partial z^2}+k^2(z)p=0 \tag{1}$$

where p is acoustic field pressure, r is the horizontal distance from a point source, z is the depth below the ocean surface, $k = 2\pi f/c = 2\pi/\lambda$ is the wave number, λ is the wavelength, $f = \omega/(2\pi)$, and ω is source frequency. Equation (1) is referred to as the elliptic-reduced wave equation. Using the reference value of 1 μPa, the propagation loss (dB) can be computed as

$$PL = 10\log(p^2)^{-1} = -20\log|p| \tag{2}$$

The geometrical assumptions and solution type chosen for p produce different canonical models. In RAM, $p = r^{-1/2}e^{ik_0 r}F(r,\theta,z)$, where $k_0 = 2\pi f/c_0$, and c_0 is the reference sound speed. Away from the source, the following far-field equation [6] holds in range-independent region,

$$\frac{\partial^2 p}{\partial r^2}+\rho\frac{\partial}{\partial z}(\frac{1}{\rho}\frac{\partial p}{\partial z})+k_1^2 p=0 \quad , \tag{3}$$

where, ρ is the fluid density, $k_1 = (1+i\eta\beta)\omega/c$ is the complex wave number, c is the sound speed, β is the attenuation (dB/λ), and η is a constant. As can be seen from eq. (3), propagation loss is a function of range (r), and depth (z), and is influenced by source frequency (f), sound speed profile, and environmental variables such as density and sea floor.

B. Background on BN Inference and Estimation

A BN model is a *directed acyclic graph* (DAG) comprised of a set of nodes representing the system variables, and a set of arcs representing relationships between the nodes [8]. In this paper, capital letters denote variables and lowercase letters denote the *states* or instantiations of the variables, e.g., X_i is said to be in its j^{th} instantiation when $X_i = x_{i,j}$. If there is an arc from A to B, B is said to be a child of A, and A is a parent of B. A *conditional probability table* (CPT) lists in tabular form the conditional probabilities of each node, X_i, $P[X_i|pa(X_i)]$, where $pa(X_i)$ denotes the parents of X_i. Let μ_i denote the instantiations of the children of X_i. Then, Bayes' rule is utilized to infer the posterior probability distribution of an unknown variable X_i, i.e., $P[X_i|\mu_i]$.

Since continuous BNs currently support only Gaussian distributions, the sonar variables are discretized and assumed finite and countable. Existing discretization methods include but are not limited to equal-width discretization (EWD), equal-frequency discretization (EFD), fuzzy discretization (FD), and lazy discretization (LD) [10]. Both EWD and EFD sort the data values of a continuous variable X_i from its minimum to its maximum values, $[x_i^{max}, x_i^{min}]$. EWD divides the number line $[x_i^{max}, x_i^{min}]$ into n intervals of equal width, w. The number of intervals is also referred to as the *instantiation number* of X_i. EFD divides the sorted values into n intervals so that each interval contains approximately the same number of training instances. Since the instantiation number n is chosen without considering the properties of the data set, both EWD and EFD may lose some attribute information. However, their advantages are that they are very simple, and often perform surprisingly well. Thus, in this paper, EWD and EFD are used to discretize the continuous variables required by the BN acoustic model presented in Section III.

A BN model can be obtained from data, by learning its structure and, then, its parameters or CPTs [11]. In this paper, the data set, O, is obtained from RAM and is complete. Thus, structural learning is performed using the greedy search algorithm known as K2 [11, 12]. For parameter learning, the *Expectation-Maximization* (EM) algorithm is used to maximize the posterior distribution $P(\Theta \mid O, G)$ with respect to the parameters, Θ, given the learned BN structure, G, and the data set, O.

III. METHODOLOGY

A. BN Acoustic Model

The universe U of the BN model is obtained from the list of input and output variables of the RAM software. A description of these variables is provided in Table 1. As can be seen from the variable instantiations (Table 1), the data for the input nodes, R, Z, SF, BD, and F, are generated uniformly. With these inputs, there are 40,000 possible instantiations for the output, PL. The non-uniformity of PL information requires careful discretization of the training and validation data used to obtain the BN model. Since the discretization technique can greatly affect the BN model performance, training is performed using both EWD and EFD, and the results are compared in Section IV. After the variables are discretized and the data set O is generated with RAM, 75% of the data is randomly selected to form the training set, T, and the remaining data, comprised of 10,000 cases, is used to form the validation set, $\mathcal{V}$. The learned BN model is shown in Fig. 2.

An important advantage of this methodology, is that after the BN model is trained, any of its nodes can be inferred given evidence from other node(s).

TABLE 1 LIST OF NODES IN BN ACOUSTIC MODEL

Node Type	Node	Instantiations and ranges
Position	Range (m): R	depends on application, e.g., [100:100:1000]
	Depth (m): Z	depends on application, e.g., [50:50:500]
Environment	Sea Floor: SF	flat; uphill; downhill; up and down
	Bottom Density (g/cc): BD	10 instantiations: [1.5:0.1:2.4],
	Bottom Sound Speed (m/s): V	quadratic experiential function of BD [13], 10 instantiations
Source	Source Frequency (HZ): F	depends on source characteristics, e.g., [10:20:200]
Output	Propagation Loss (dB): PL	depends on discretization method, 10 or 20 instantiations

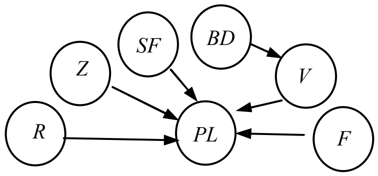

Fig. 2. Learned structure of BN acoustic-wave propagation model.

B. BN Performance Metrics

The outcome of an inferred variable $X_i \in U$ is a posterior probability distribution over its domain, $dom(X_i)$, representing the set of all of its possible instantiations (Table 1). The instantiation with the maximum posterior probability is typically chosen as the estimate for X_i, based on the available evidence e, i.e.:

$$\hat{x} = \underset{x_{i,j} \in\ dom(X_i)}{\arg\max} \{P(X_i = x_{i,j} \mid e)\} \qquad (4)$$

The *confidence level* (CL) of the estimate is the maximum posterior probability, $P(X_i = \hat{x} \mid e)$, and is a measure of the BN confidence in the accuracy of the value $\hat{x}$. Let the *classification accuracy* (CA) be a Boolean metric that is equal to one when the estimate $\hat{x}$ is correct and is equal to zero otherwise. The CA metric has the following disadvantages, it neglects CL and does not account for the discretization error.

We introduce the expected distance metric (EDM) to overcome the disadvantages of CA. The discretization induces the error interval I_j, $j = 1, \ldots, n$, where n is the number of variable instantiations, $I_j = [x_j^{min}, x_j^{max}]$, and $\bar{x}_j \equiv (x_j^{min} + x_j^{max}) / 2$. Let x_i^* denote the true value of X_i, which could be available from either T or $\mathcal{V}$, depending on whether the BN is being trained or validated. Then, the EDM of the estimate for X_i is

$$\text{EDM} \equiv \left| \, x_i^* - \sum_{x_{i,j} \in dom(X_i)} \bar{x}_j \cdot P(X_i = x_{i,j} \mid e)\} \, \right| \qquad (5)$$

As can be seen from the above definition, EDM accounts for CL and has the same units as X_i, in the continuous domain. The smaller is the value of EDM, the higher is the accuracy of the BN estimate for X_i.

IV. Results

The BN model performance over the validation data $\mathcal{V}$, which is not used for training, is illustrated in Tables 2-3. The BN is used to infer variables *PL*, *Z*, and *R* under a wide variety of environmental conditions, and its estimates are compared to the *true* values obtained by RAM. As can be seen from Tables 2-3, the results show that in this application EWD generates training data more effectively than EFD. The BN acoustic model learned by EWD predicts the *PL* with an accuracy ranging from 84.6% to 94.8%, as shown in Table 2. Also, as an example, the spatial distribution of *PL* predicted by the BN (Fig. 3.b) is compared to the PE model output in Fig. 3.a, leading to the EDM error shown in Fig. 3.c. The resulting BN model is found to outperform an equivalent neural network model implementing radial basis functions (see Table 4), and can be easily integrated in sonar control algorithms, such as in [14].

TABLE 2 Average CA Metric for Different Discretization Methods and Inferred Variables (Output)

Discretization Method	EWD		EFD	
Number of *PL* instantiations, *n*	10	20	10	20
Output: *PL*	0.95	0.85	0.60	0.48
Output: *Z*	0.17	0.28	0.41	0.49
Output: *R*	0.17	0.27	0.42	0.51
Outputs: *R* & *Z*	0.14(*Z*) 0.14(*R*)	0.19(*Z*) 0.19(*R*)	0.25(*Z*) 0.24(*R*)	0.28(*Z*) 0.28(*R*)

TABLE 3 Average EDM Metric for Different Discretization Methods and Inferred Variables (Output)

Discretization Method	EWD		EFD	
Number of *PL* instantiations, *n*	10	20	10	20
Output: *PL* [dB]	6.52	5.70	14.31	8.08
Output: *Z* [m]	104.72	93.90	82.34	73.43
Output: *R* [m]	223.65	189.81	147.65	129.39
Outputs: *R* & *Z*	111.13(*Z*) 234.38(*R*)	103.95(*Z*) 208.16(*R*)	99.75(*Z*) 183.63(*R*)	96.92(*Z*) 178.80(*R*)

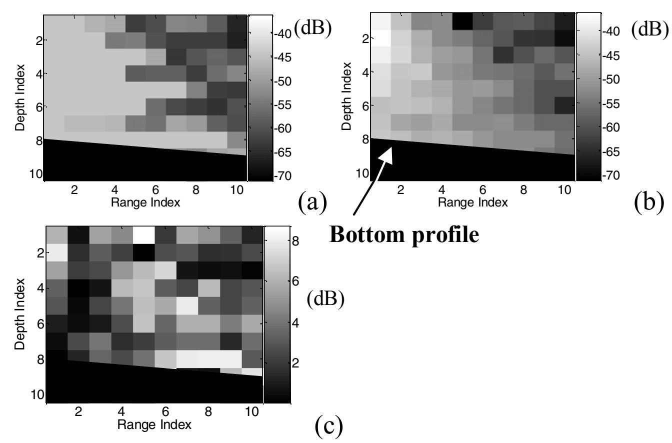

Fig. 3: An example of BN prediction of *PL* from RAM (a), the BN model (b), and absolute error between the two (c).

TABLE 4 Comparison of Bayesian and Neural Network Modeling

Method:	**BN Modeling**				**RBN Modeling**	
Discretization:	EWD		EFD		EWD	
Number of *PL* instantiations, *n*	10	20	10	20	10	20
PL Prediction CA (%):	95%	85%	60%	48%	82%	75%

The maximum detection range of an acoustic sensor is known to be influenced by its environmental conditions, because the latter influence propagation loss. Hence, the environment characteristics that influence sound-wave propagation in the ocean (Table 1), also influence the sensor performance. Since sonar control strategies require an estimate of maximum sensor range [14,15], the BN model is used here to determine this range as a function of the sensor position within an area of interest.

It is assumed that all parameters and conditions are time invariant. The target strength (*TS*) and detection threshold (*DT*) are assumed as given and are location invariant. The noise level (*NL*) over the area is assumed to have a Gaussian distribution, a range [66, 78] dB, and is estimated using the data on average deep-water ambient-noise spectra in [16], based on moderate-to-heavy ship traffic. According to the passive sonar equation,

$$SL + DI_s - PL - (NL - DI) = DT \quad (6)$$

the maximum value of *PL* leading to a detection above the threshold can be determined for a sensor with known directivity index (*DI*) and target-source directivity (DI_s). Subsequently, this value of *PL* together with any available environmental conditions can be provided as evidence to the BN model to infer the maximum value of the range variable, R_{max}. A plot of this range with respect to the latitude and longitude coordinates is shown in Fig. 4. It can be seen that, within the same area of interest, the sensor performance can vary significantly and in a highly nonlinear fashion. The range obtained by the BN as a function of sensor position is of great value to algorithms for control and sensor placement [14].

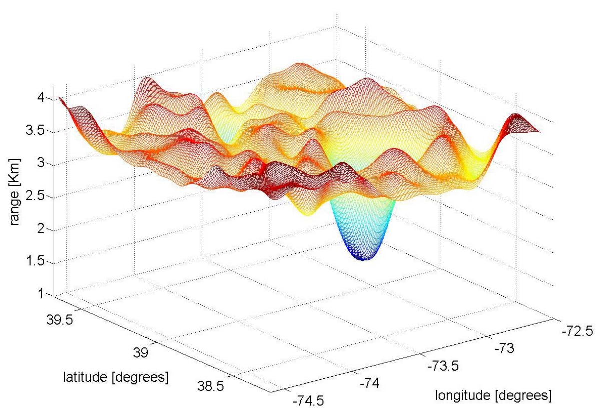

Fig. 4. Sensor detection range over an oceanic area of interest.

V. Conclusion and Future Work

A Bayesian network model of acoustic-wave propagation is presented for use in sonar control applications. The BN can be used to predict propagation loss when a compromise between modeling accuracy and computation burden must be achieved. Unlike high-fidelity acoustic models based on numerical integration of PDEs, the BN model provides a closed-form representation of the process and can be used to infer any variables in its universe. In order to achieve better accuracy, two different continuous variable discretization methods, EWD and EFD, are compared. The BN model is shown to outperform a radial basis neural network trained with the same data, and to provide reasonable accuracy over an extensive validation set. In addition, the BN model is combined with the sonar equation to estimate the maximum range of an acoustic sensor as a function of its position in an oceanic area of interest. Consequently, the BN is of great value to algorithms aimed at placing and deploying sonar for coverage, target detection, classification, and tracking.

Acknowledgment

The authors wish to thank Thomas Wettergren, Warren Fox, Casey Rubin, and Kelli Baumgartner for their useful comments and suggestions. This work was sponsored by the Office of Naval Research Young Investigator Program (321).

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