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On the Duality of Robot and Sensor Path Planning

Ashleigh Swingler and Silvia Ferrari

Abstract— **The performance of a mobile sensor can be greatly improved by planning its path with respect to its sensing objective, field-of-view, and platform geometry. Although many algorithms have been developed for the related field of robot path planning, a majority of these methodologies cannot be directly applied to the problem of sensor path planning. This paper presents a technique by which mixed-integer programming (MIP) can be used to determine the optimal path of a mobile sensor. MIP is able to return solutions in non-convex environments, and has a flexible framework that allows for the consideration of vehicle dynamics, obstacle avoidance, and, as shown here, target measurement objectives. The primary contribution of this work is the development of a poof of the duality of robot and sensor path planning. By use of MIP, the proof shows that many approaches to classical robot navigation problems can be reformulated for sensor path planning. Illustrative simulation results for the paths of mobile robots and sensor platforms are presented; MATLAB and Tomlab/CPLEX were used to solve the path optimization problems.**

I. INTRODUCTION

The field of sensor path planning is interested in determining the optimal path of a sensor, installed on a mobile robotic platform, that both maximizes the information profit and minimizes the performance cost of the sensing system. Finding a path that supports a primary measurement objective, for example - target classification, distinguishes sensor path planning problems from traditional robot path planning problems. The latter addresses purely navigational objectives, an example of which is collision avoidance in unstructured, dynamic environments. Furthermore, path planning techniques for mobile robots generally aim to optimize deterministic additive functions, while sensor path planning approaches look to optimize stochastic sensing objectives, which are not necessarily additive. As a result of the aforementioned differences, many of the methodologies proposed for robot path planning cannot be directly applied to the problem of sensor path planning. The geometry of the field-of-view (FOV) of a mobile sensor is an important consideration in planning its optimal path in an environment. The FOV refers to the bounded space in which the sensor is able to make measurements. In order for a target to be observable its geometry must intersect that of the FOV of the sensor. Therefore, the geometry and positions of the targets must also be accounted for in sensor path planning methodologies.

Recently, many traditional robot path planning techniques, including cell decomposition [1], [2], artificial potential fields [3], [4], optimal control [5], and randomized hybrid methods [6], have been successfully extended to solve sensor planning problems. Artificial potential field approaches are well-established in the field of robot path planning as they are suited for online planning in the presence of obstacles. These methods, however, are subject to local minima, and are often unable to account for the positions and geometries of targets as well as the FOV of the sensor. In order to circumvent these limitations, the authors in [4] proposed a novel approach for sensor planning using artificial potential fields in which attractive potentials are generated for the targets, and by use of a potential function, a probabilistic roadmap is constructed, such that the sensor is able to escape local minima. An inherent disadvantage of potential field methods is that they are limited to local solutions, and are, therefore, unable to guarantee globally optimal sensor paths. Cell decomposition, on the other hand, is able to return globally optimal solutions in path planning problems. In [1], approximate cell decomposition (ACD) was adapted for establishing the optimal measurement strategy of a mobile sensing platform, with a bounded FOV, in an obstacle populated environment. The approach developed has the advantage that it can effectively take into account the geometry and FOV of the sensor, and the geometries and positions of both the targets and the obstacles in the workspace, as compared to alternate sensor planning methods. Unfortunately, obtaining an ACD of an environment can be computationally expensive, and the techniques available cannot readily account for kinematic and dynamic motion constraints.

Mixed integer programming (MIP), and its mathematical equivalent, disjunctive programming, are methodologies used for the inclusion of both discrete and continuous constraints in optimization problems [7]. In recent years, MIP has been demonstrated to be a successful technique for determining collision-free paths for robotic vehicles in obstacle populated environments [8]–[10]. Although MIP can be computationally intensive, it is able to return globally optimal, complete solutions, and has a flexible framework that allows for the consideration of a variety of planning constraints, including multi-agent safety guarantees [11]–[13], holonomic and non-holonomic kinematic constraints [14]–[16], and communication and control bounds [17]. In this paper a methodology is developed for sensor path planning using MIP. Furthermore, by formulating the sensor path planning problem as a MIP it can be shown to be a mathematical dual of the robot path planning problem.

The remainder of the paper is organized as follows: The problem formulation and assumptions of the classic robot

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A. Swingler is a graduate student in Mechanical Engineering, Duke University, Durham NC, USA ashleigh.swingler@gmail.com

S. Ferrari is with the Faculty of Mechanical Engineering, Duke University, Durham NC, USA sferrari@duke.edu

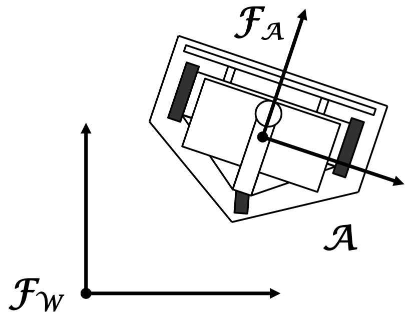


Fig. 1. Geometry of a robot, $\mathcal{A}$, in a workspace, $\mathcal{W}$

path planning problem are discussed in Section II. A MIP approach to obstacle avoidance for a mobile robot is outlined in Section III. Section IV discusses the problem of mobile sensor path planning. The dual MIP formulation of the sensor path planning problem is introduced in Section V. Simulation results are presented in Section VI. Conclusions are found in Section VII.

II. ROBOT PATH PLANNING PROBLEM FORMULATION

Robot path planning methodologies seek to determine obstacle-free paths of minimum cost for a mobile robot, between two configurations in a region-of-interest (ROI) or workspace, $\mathcal{W} \subset \mathbb{R}^N$. The robot is assumed to have a rigid, convex geometry that is a compact subset of the workspace and is denoted $\mathcal{A} \subset \mathcal{W}$. Let $\mathcal{F}_{\mathcal{W}}$ represent a fixed, Cartesian reference frame embedded in $\mathcal{W}$, and let $\mathcal{F}_{\mathcal{A}}$ denote a moving, Cartesian reference frame embedded in $\mathcal{A}$, as shown in Fig. 1. In this paper, the ROI is assumed to be populated by n obstacles, $\mathcal{B}_1, \ldots, \mathcal{B}_n$, that are stationary with respect to $\mathcal{F}_{\mathcal{W}}$, and have coordinates and geometries that are known *a priori*. Although not addressed in this analysis, it is also possible to consider moving obstacles and obstacles that are detected in real time by on-board sensors [3], [5]. The robot configuration, $\mathbf{q}$, defines the position and orientation of $\mathcal{F}_{\mathcal{A}}$ in $\mathcal{F}_{\mathcal{W}}$, and can be used to specify the location of every point contained in $\mathcal{A}$. The subset of the workspace occupied by the geometry of the robot with configuration $\mathbf{q}$ is denoted $\mathcal{A}(\mathbf{q})$.

In many planning techniques it is assumed that the robot, in the absence of obstacles, is able to follow any feasible path. This assumption is referred to as a free-flying model, in which the motion of the robot is not constrained by any kinematic or dynamic constraints [18]. The latter greatly simplifies path planning techniques as the motion of the robot is only limited by the presence of obstacles; however, this model does not provide a practical representation of robot maneuvering. Therefore, most robot planning analysis techniques assume either a dynamic or kinematic model for the motion of the platform. A kinematic model, for example the unicycle or car-like models [19], implicitly define the feasible trajectories of a robot, and can contain motion constraints - such as the no-slip condition, and physical limitations on the controls of the robot - such as a minimum turning radius. Recently, several authors have considered computing the optimal path of a robot subject to a set of dynamic constraints [2], [3]. Dynamic models describe the influence of external forces, moments, torques, and control inputs on the motion of the vehicle, and provide realistic constraints for robot motion planning. In this paper, as formulated below, the problem of determining a minimum cost collision-free path subject to the dynamic motion constraints of a robot with geometry $\mathcal{A}$ is considered,

$$\begin{aligned} \min_{\mathbf{q},\mathbf{u}} J_c = \min_{\mathbf{q},\mathbf{u}} \int_0^{\infty} \mathcal{L}\left(\mathbf{q}, \mathbf{u}, t\right) \mathrm{d}t \\ \text{subject to} \quad \dot{\mathbf{q}}(t) = f\left(\mathbf{q}, \mathbf{u}, t\right) \end{aligned} \tag{1}$$

where $\mathbf{u}$ represents the controllable inputs to the robot and t is time. In the presence of obstacles, the optimization problem in (1) is inherently non-convex and, as a result, can be challenging for many algorithms to solve. MIP, which employs both integer and continuous variables, can be used to effectively determine solutions to path planning problem, by modeling collision avoidance constraints as discrete decisions, as outlined in Section III.

In order to solve the robot path planning problem numerically, a finite horizon time, t_{F}, is enforced. The finite horizon time is divided into $T = \frac{t_{\mathrm{F}}}{\Delta t}$ time-steps of length Δt. In this paper, it is assumed that the discrete kinematics of the robot can be represented by a linear relationship and the discrete cost can be represented by a quadratic relationship, both functions of the state and control vectors of the robot. Then, the robot path planning optimization problem in (1) can be approximated by the quadratic program below,

$$\begin{aligned} \min_{\mathbf{q},\mathbf{u}} J = \min \sum_{k=0}^{T} \left(\mathbf{q}^{\mathrm{T}}(k)\mathbf{P}\mathbf{q}(k) + \mathbf{u}^{\mathrm{T}}(k)\mathbf{R}\mathbf{u}(k)\right) \\ \text{subject to} \quad \forall k \in [0, \ldots, T-1] \\ \mathbf{q}(k+1) = \mathbf{F}\mathbf{q}(k) + \mathbf{G}\mathbf{u}(k) \\ \mathbf{q}(k=0) = \mathbf{q}_0 \\ \mathbf{q}(k=T) = \mathbf{q}_{\mathrm{F}} \end{aligned} \tag{2}$$

where $\mathbf{P}$ and $\mathbf{Q}$ are (semi-) positive definite weighting matrices, and $\mathbf{q}_0$ and $\mathbf{q}_{\mathrm{F}}$ are the initial and final configurations of the robot, respectively. The above formulation can be used to represent a variety of path optimization scenarios including determining the path of minimum distance or the path of minimum-energy consumption.

III. MIP FOR ROBOT PATH PLANNING

The configuration space, denoted $\mathcal{C}$, defines the space of all possible configurations of the robot. Let the C-obstacle $\mathcal{CB}_i$ represent a mapping of the obstacle $\mathcal{B}_i$ from the workspace to the configuration space,

$$\mathcal{CB}_i \equiv \{\mathbf{q} \in \mathcal{C} | \mathcal{A}(\mathbf{q}) \cap \mathcal{B}_i \neq \emptyset\} \tag{3}$$

which can be calculated by use of Minkowski summations [18]. Consider the robot found in Fig. 2(a); assuming that the robot is only capable of translation and not rotation, the

C-obstacle associated with the obstacle found in Fig. 2(b) can be determined by sliding the geometry of $\mathcal{A}$ along the boundary of the obstacles, as exhibited in Fig. 2(c).

For a fixed robot orientation and assuming that the geometries of both the robot and the obstacles are polyhedral, the consequent C-obstacle will also be polyhedral in shape. An s-sided polygon can be represented by s straight line segments, an example of which is shown in Fig. 3. A robot will avoid collisions with any obstacle $\mathcal{B}_i$ with corresponding s_i-sided C-obstacle $\mathcal{CB}_i$, provided the following condition is satisfied at all time steps $k \in [0, \ldots, T]$,

$$\bigvee_{j=1,\ldots,s_i} \mathbf{a}_{ij}^{\mathrm{T}} \mathbf{q}(k) \geq b_{ij} \tag{4}$$

where $\mathbf{a}_{ij}^{\mathrm{T}}$ and b_{ij} are defined for $\mathcal{CB}_i$ as in Fig. 3.

By implementing the so-called "big-M" technique, which is used to connect logical indicator variables to continuous variables [17], [20], the logical disjunctions in (4) can be converted into the linear mixed integer constraints below,

$$\begin{aligned} &\mathbf{a}_{i1}^{\mathrm{T}} \mathbf{q}(k) \geq b_{i1} - M w_{i1}(k) \\ &\mathbf{a}_{i2}^{\mathrm{T}} \mathbf{q}(k) \geq b_{i2} - M w_{i2}(k) \\ &\quad \vdots \\ &\mathbf{a}_{is_i}^{\mathrm{T}} \mathbf{q}(k) \geq b_{is_i} - M w_{is_i}(k) \\ &\sum_{j=1}^{s_i} w_{ij}(k) \leq (s_i - 1) \end{aligned} \tag{5}$$

where $w_{ij}(k)$ is a binary slack variable, such that $w_{ij}(k) = 0$ or 1, and M is a large positive integer. Let,

$$\mathbf{w}_i(k) = [w_{i1}(k)\ w_{i2}(k)\ \ldots\ w_{is_i}(k)]^T \tag{6}$$

$$\mathbf{A}_i = [\mathbf{a}_{i1}\ \mathbf{a}_{i2}\ \ldots\ \mathbf{a}_{is_i}]^T \tag{7}$$

$$\mathbf{b}_i = [b_{i1}\ b_{i2}\ \ldots\ b_{is_i}]^T \tag{8}$$

By use of the definitions above the constraints in (5) can be written in matrix notation,

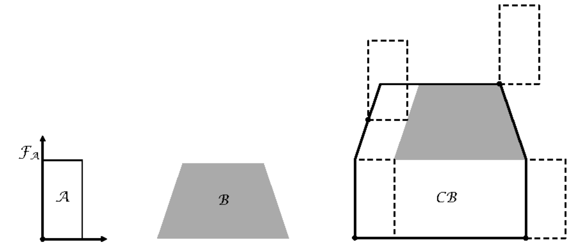
$$\begin{aligned} &\mathbf{A}_i \mathbf{q}(k) + M \mathbf{w}_i(k) \geq \mathbf{b}_i \\ &\boldsymbol{\alpha}_i^{\mathrm{T}} \mathbf{w}_i(k) \leq (s_i - 1) \\ &w_{ij} = 0 \text{ or } 1 \quad \forall j \in [1, \ldots, s_i] \end{aligned} \tag{9}$$

where $\boldsymbol{\alpha}_i$ is a vector of ones with dimension $s_i \times 1$. The set of equations in (9) guarantees that the robot will avoid collisions with obstacle $\mathcal{B}_i$.

Based on the above assumptions, the optimal, collision-free trajectory for a robot deployed in an environment containing n polygonal obstacles, can be determined by solving the MIP below for the control vector $\mathbf{u}$,

$$\begin{aligned} &\min_{\mathbf{q},\mathbf{u}} J = \min \sum_{k=0}^{T} \left(\mathbf{q}^{\mathrm{T}}(k) \mathbf{P} \mathbf{q}(k) + \mathbf{u}^{\mathrm{T}}(k) \mathbf{R} \mathbf{u}(k) \right) \\ &\text{subject to} \quad \forall k \in [0, \ldots, T-1] \\ &\qquad \mathbf{q}(k+1) = \mathbf{F}\mathbf{q}(k) + \mathbf{G}\mathbf{u}(k) \\ &\qquad \mathbf{q}(k=0) = \mathbf{q}_0 \\ &\qquad \mathbf{q}(k=T) = \mathbf{q}_{\mathrm{F}} \\ &\qquad \forall k \in [0, \ldots, T],\ \forall i \in [1, \ldots, n] \\ &\qquad \mathbf{A}_i \mathbf{q}(k) + M \mathbf{w}_i(k) \geq \mathbf{b}_i \\ &\qquad \boldsymbol{\alpha}_i^{\mathrm{T}} \mathbf{w}_i(k) \leq s_i - 1 \\ &\qquad w_{ij}(k) = 0 \text{ or } 1 \quad \forall j \in [1, \ldots, s_i] \end{aligned} \tag{10}$$

The above MIP can be readily implemented using the commercially available software programs MATLAB and Tomlab/CPLEX.



(a) Robot, $\mathcal{A}$ (b) Obstacle, $\mathcal{B}$ (c) C-obstacle, $\mathcal{CB}$

Fig. 2. Determination of C-obstacle $\mathcal{CB}$ for robot $\mathcal{A}$ and obstacle $\mathcal{B}$

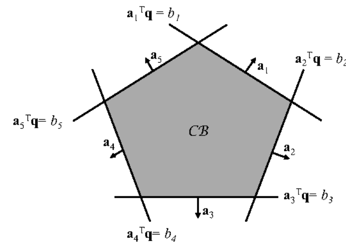


Fig. 3. Obstacle avoidance constraints as defined by C-obstacle $\mathcal{CB}$

IV. SENSOR PATH PLANNING PROBLEM FORMULATION

Mobile sensors have recently emerged as effective and inexpensive solutions to a variety of dynamic measurement problems, including the monitoring of urban environments, the localization of fugitive pollutant emissions, and military reconnaissance missions. The success of these systems can be attributed to modern advancements in the optimal control and automation of mobile sensors and sensor networks. Sensor path planning problems seek to determine obstacle free paths with maximum information value between two configurations in a workspace $\mathcal{W} \subset \mathbb{R}^N$. Here, the problem studied is such that the sensor is deployed in order to obtain measurements from m fixed targets, $\mathcal{T}_1, \ldots, \mathcal{T}_m$, in $\mathcal{W}$. For simplicity, this paper assumes that the positions and

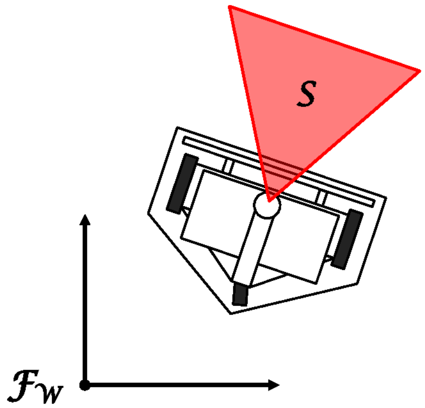


Fig. 4. Geometry of the sensor

geometries of all targets are known *a priori*, and all targets are polyhedral.

In the scenario considered, the sensor is installed on a robotic platform with geometry $\mathcal{A}$, such that the position and orientation of the field-of-view (FOV) of the sensor, $\mathcal{S} \in \mathbb{R}^N$, are fixed with respect to $\mathcal{A}$, as illustrated in Fig. 4. Moreover, it is assumed that the geometries of both $\mathcal{A}$ and $\mathcal{S}$ can be represented by rigid polygons. Let $\mathcal{F}_\mathcal{A}$ denote a moving, Cartesian reference frame embedded in the sensing platform, and $\mathcal{F}_\mathcal{W}$ defines a fixed, Cartesian reference frame embedded in $\mathcal{W}$. The position and orientation of $\mathcal{F}_\mathcal{A}$ in $\mathcal{F}_\mathcal{W}$ are defined by the configuration $\mathbf{q}$.

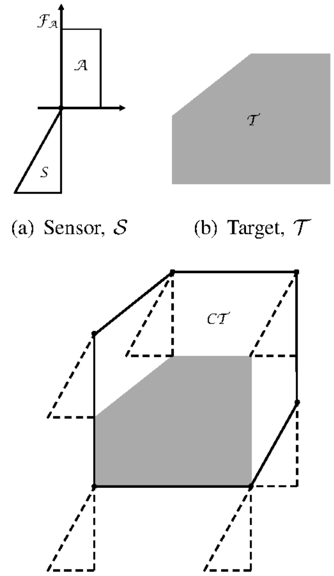
The primary objective of the mobile sensing system, in which the sensor must gather measurements from each of the m targets located in $\mathcal{W}$, can be formulated as a set of mixed integer constraints, as discussed in the following Section. In addition, the path determined must minimize the operational cost of the mobile platform, subject to the system's dynamic constraints. Therefore, the sensor path planning problem can be defined as in (1). In order to determine the optimal path of the sensor numerically, as in Section II, the optimization problem is discretized with respect to time. The dynamics of the sensor platform are assumed to be linear functions of the configuration and controls of the sensor. Therefore, the optimal path of the sensor can be determined by solving the system in (2) for $\mathbf{u}$.

V. DUALITY OF ROBOT AND SENSOR PLANNING PROBLEMS

The FOV of a sensor defines the finite subset of the workspace, with geometry $\mathcal{S}$, in which the sensor is able to make measurements. A target located at a point $p \in \mathcal{S}(\mathbf{q})$ is measured by the sensor when the platform has configuration $\mathbf{q} \in \mathcal{C}$. A *C-target* $\mathcal{CT}_i$ is defined as a mapping of a target $\mathcal{T}_i$ from $\mathcal{W}$ to the configuration space of the robotic sensor [1],

$$\mathcal{CT}_i = \{\mathbf{q} \in \mathcal{C} | \mathcal{S}(\mathbf{q}) \cap \mathcal{T}_i \neq \emptyset\} \tag{11}$$

The relationship between the FOV of a sensor, a target, and the associated C-target is exhibited in Fig. 5. Under the assumption that the geometries of both the FOV of the sensor and the targets are polyhedral, for a fixed orientation angle, a target $\mathcal{T}_i$ is assumed to have an r_i-sided polygonal C-target that can be defined by r_i straight line segments, as in Fig. 5. A measurement of target $\mathcal{T}_i$ occurs when the FOV of the sensor intersects the geometry of the target,



(a) Sensor, $\mathcal{S}$ (b) Target, $\mathcal{T}$

(c) C-Target, $\mathcal{CT}$

Fig. 5. Determination of the C-target found in (c) for the sensor with FOV geometry in (a) and the target in (b)

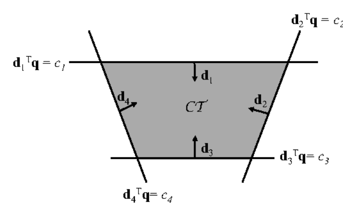


Fig. 6. Target Measurement Constraints

$$\begin{aligned} \mathbf{d}_{i1}^T \mathbf{q}(k) &\leq c_{i1} \\ \mathbf{d}_{i2}^T \mathbf{q}(k) &\leq c_{i2} \\ &\vdots \\ \mathbf{d}_{ir_i}^T \mathbf{q}(k) &\leq c_{ir_i} \end{aligned} \tag{12}$$

The constraints in (12) can be represented by the matrix-vector equation,

$$\mathbf{D}_i \mathbf{q}(k) \leq \mathbf{c}_i \tag{13}$$

where,

$$\mathbf{D}_i = [\mathbf{d}_{i1} \ \mathbf{d}_{i2} \ \ldots \ \mathbf{d}_{ir_i}]^T \tag{14}$$

$$\mathbf{c}_i = [c_{i1} \ c_{i2} \ \ldots \ c_{ir_i}]^T \tag{15}$$

Equation (13) guarantees that at time step k the sensor is able to measure target $\mathcal{T}_i$. However, in order to satisfy the sensing objective of the path planning problem, all m targets must be measured. Let $z_i(k)$ be a binary slack variable. Then, using the big-M technique, as in Section III, the sensing objective is accomplished when the following is satisfied for all $i \in [1, \ldots, m]$,

$$\begin{aligned} &\mathbf{D}_i \mathbf{q}(k) \leq \mathbf{c}_i + M z_i(k) \\ &\sum_{k=0}^{T} z_i(k) \leq T \\ &z_i = 0 \text{ or } 1 \end{aligned} \tag{16}$$

Define $\boldsymbol{\beta}_i$ as a vector of ones of dimension $(T+1)\times 1$ and the vector $\mathbf{z}_i$ as follows,

$$\mathbf{z}_i = [z_i(k=0)\ z_i(k=1)\ \ldots\ z_i(k=T)]^T \tag{17}$$

Then, by introducing (16) and (17) into the sensor path optimization problem defined in (2) and solving for $\mathbf{u}$, it is possible to determine a control sequence that results in the sensor following a path of minimum cost that fulfills the sensing objective, and is feasible with respect to the dynamic constraints of the platform. The final MIP of the sensor planning problem takes the form,

$$\begin{aligned} &\min_{\mathbf{q},\mathbf{u}} J = \min \sum_{k=0}^{T} (\mathbf{q}^{\mathrm{T}}(k)\mathbf{P}\mathbf{q}(k) + \mathbf{u}^{\mathrm{T}}(k)\mathbf{R}\mathbf{u}(k)) \\ &\text{subject to} \quad \forall k \in [0,\ldots,T-1] \\ &\qquad \mathbf{q}(k+1) = \mathbf{F}\mathbf{q}(k) + \mathbf{G}\mathbf{u}(k) \\ &\quad \mathbf{q}(k=0) = \mathbf{q}_0 \\ &\quad \mathbf{q}(k=T) = \mathbf{q}_F \\ &\quad \forall k \in [0,\ldots,T],\ \forall i \in [1,\ldots,m] \\ &\qquad -\mathbf{D}_i\mathbf{q}(k) + M z_i(k) \geq -\mathbf{c}_i \\ &\qquad \boldsymbol{\beta}_i^{\mathrm{T}} \mathbf{z}_i \leq T \\ &\qquad z_i(k) = 0 \text{ or } 1 \end{aligned} \tag{18}$$

By inspection, the MIP above is in the same form as (10), which is used to solve the robot path planning problem for collision avoidance. Therefore, the sensor path planning problem can be considered a mathematical dual of the robot path planning problem.

VI. SIMULATIONS

Simulation results are presented for three fundamental scenarios, robot path planning for obstacle avoidance, sensor path planning for target measurement, and sensor path planning in the presence of both targets and obstacles. For simplicity, the motions of the robot and the mobile sensor are limited to translation and are constrained by double integrator dynamics, however, it is possible to extend the MIP methodology to problems involving rotations and non-holonomic dynamics. The simulations selected are intended to clearly illustrate the duality of sensor path planning and robot path planning problems. The first example considers the problem of a translating robot that must navigate an obstacle populated environment between two configurations, $\mathbf{q}_0$ and $\mathbf{q}_{\mathrm{F}}$. By use of the methodology outlined in Section III for path planning in the presence of obstacles, MIP is able

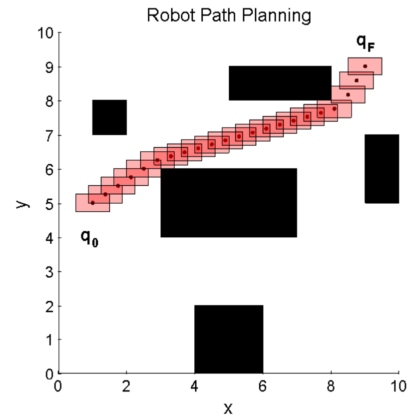


Fig. 7. Simulation results for robot path planning problem. Robot is indicated in red and obstacles are marked in black.

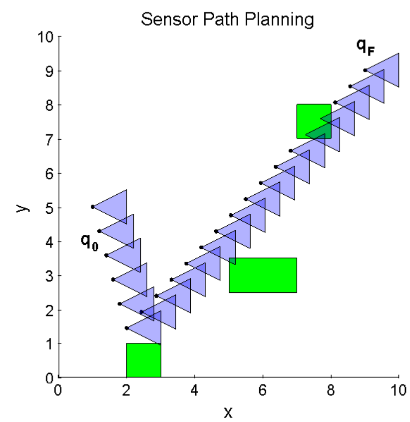


Fig. 8. Simulation results for sensor path planning problem. Targets are indicated in green while the FOV of the sensor is marked in blue.

to return a collision free path of minimum cost, in this case minimum distance, as exhibited by the simulation in Fig. 7.

The novel approach developed in this paper addresses the problem of optimal sensor path planning, by extending previous work in MIP for robot path planning. Unlike the latter, the objective of sensor path planning is to fulfill a sensing objective. In the scenario considered, the mobile sensor must obtain measurements from each of the targets located in the workspace, along a pah of minimum distance. The sensor is a assumed to be a point mass with a convex FOV. As outlined in Section V, by modeling the sensor planning problem as a dual of the robot path planning problem it is possible to determine the path of the sensor, the results of which are found in Fig. 8. This simulation illustrates that MIP can be effectively implemented in sensor path planning problems in order to represent the measurement objective of the sensor.

In most realistic sensor path plannings scenarios, the geometry of the mobile platform, which the sensor is installed on, must be taken into consideration. Fig. 9 exhibits that by combining the MIP techniques discussed, it is possible to determine a path of minimum cost that both fulfills the sensing objective of the sensor, while avoiding collisions

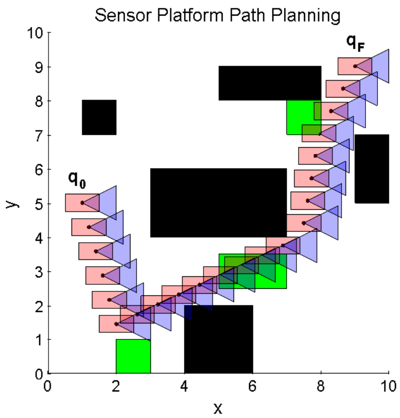


Fig. 9. Sensor platform path planning problem in the presence of targets and obstacles.

between the sensor platform and the workspace obstacles.

VII. CONCLUSIONS AND FUTURE WORK

This paper presents a proof for the duality of robot and sensor path planning, and reviews previous work on MIP for robot path planning. Sensor path planning, as formulated here, is shown to be a mathematical dual of the classic robot path planning. The proposed MIP methodology has the advantage over other sensor planing techniques in that it returns globally optimal, complete solutions. Furthermore, the flexibility of the MIP framework can readily address problems including complex vehicle dynamics, multi-agent systems, and router connectivity. In future work, simulations will be conducted for a variety of sensing systems, including those with nonholonomic dynamics. The methodology developed will be implemented in environmental sensing scenarios that must account for the expected information value of the measurements obtained by the mobile sensor.

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