A Blade Element Approach to Modeling Aerodynamic Flight of an Insect-scale Robot

Taylor S. Clawson, Sawyer B. Fuller
Robert J. Wood, Silvia Ferrari

American Control Conference
Seattle, WA
May 25, 2016
Introduction
RoboBee Background

- Wing stroke angle $\phi_w$ controlled independently for each wing
- Thrust and body torques controlled by modulating stroke angle commands

Pitch

Roll

$\phi_w$

100 mg

14 mm

120 Hz

Video of RoboBee test flight courtesy of the Harvard Microrobotics Lab

Image Credit: [Ma K.Y., ’12], [Ma K.Y., ’13]
Introduction and Motivation

• Applications
  – Navigation in cluttered environments, requiring precise reference tracking
  – Robust stabilization, subject to large disturbances such as winds and gusts

• Research Goals
  – Control design, implementation, and guarantees
  – Develop high-fidelity simulation tools

• Previous work
  – Simplified RoboBee Flight Model [Fuller, S.B. ’14], [Chirarattananon, P. ’16]
    • 6 DOF body motion, no wing modeling
    • Linearized, uncoupled, stroke-averaged aerodynamic forces
    • Controlled with hierarchical PID and iterative learning
  – RoboBee Wing Aerodynamics [Whitney, J.P. ’10], [Jafferis, N.T. ’16]
    • Model wing aerodynamics with blade-element theory
    • Omit body dynamics (constant body position and orientation)
Blade-Element Overview

• Wing is divided span-wise into rigid 2D differential elements
• Differential forces are computed for each element, and then integrated along wingspan for total force on wing
• Tuned to provide close approximation of actual forces in an expression that is:
  – Closed-form
  – Computationally-efficient
  – Provides insight into dominant underlying physics
Model Description
Modeling Setup

- **3 Rigid bodies**
  - Main body + two wings
- **8 DOF model**
  - Main body: 6 DOF
  - Wings: 1 DOF each (pitch angle $\psi_w$)
    - Stroke angle $\phi_w$ treated as an input
    - No stroke-plane deviation $\theta_w$

### Wing Euler Angles

<table>
<thead>
<tr>
<th>Stroke Angle</th>
<th>Stroke-Plane Deviation</th>
<th>Wing Pitch</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_w$</td>
<td>$\theta_w$</td>
<td>$\psi_w$</td>
</tr>
</tbody>
</table>

**Fixed frame to body frame**

$\mathcal{F} \{ \hat{i}, \hat{j}, \hat{k} \} \Rightarrow \mathcal{B} \{ \hat{x}, \hat{y}, \hat{z} \}$

**Similar for:**

- $\mathcal{B}$ to left wing $\mathcal{L} \{ \hat{x}_l, \hat{y}_l, \hat{z}_l \}$
- $\mathcal{B}$ to right wing $\mathcal{R} \{ \hat{x}_r, \hat{y}_r, \hat{z}_r \}$
States and Inputs

\[
x = \begin{bmatrix} \Theta^T & r^T & \Theta^T_r & \Theta^T_l & \dot{\Theta}^T & \dot{r}^T & \dot{\Theta}^T_r & \dot{\Theta}^T_l \end{bmatrix}^T
\]
\[
u = [\phi_0 \ \phi_p \ \phi_r]^T
\]

Stroke angle trajectory \( \phi_w \) modeled as a function of input \( u \) following linear second-order system:

\[
\ddot{\phi}_w(t) + 2\zeta \omega_n \dot{\phi}_w(t) + \omega_n^2 \phi_w(t) = A_w \sin(\omega_f t) + \bar{\phi}_w
\]

For the right wing, for example,

\[
A_w = \phi_0 - \frac{\phi_r}{2}, \quad \bar{\phi}_w = -\phi_p
\]
Rigid Body Dynamics

- Angular momentum balance about body CG:
  \[
  \sum M_G = \sum \dot{H}_G
  \]
  \[
  \sum M_G^L + \sum M_G^R = \dot{H}_G^B + \dot{H}_G^L + \dot{H}_G^R
  \]

- Blade-element theory used to calculate aerodynamic forces and moments

- Aerodynamic forces act at instantaneous centers of pressure $CP_L$, $CP_R$
  \[
  \sum M_G^L = M_{rd}^L + r_{CP_L/G} \times F_{aero}^L + r_{L/G} \times m_L g
  \]

- Angular momentum about $G$ calculated as a sum of contributions from each frame
  \[
  \dot{H}_G^B = I^B \dot{\omega}_B + \omega_B \times I^B \omega_B
  \]
  \[
  \dot{H}_G^L = I^L \dot{\omega}_L + \omega_L \times I^L \omega_L + r_{L/G} \times m_L a_L
  \]
Wing Rigid Body Dynamics

- Single DOF: wing pitch $\psi_w$
  - Angular momentum balance in span-wise direction

$$\hat{y}_r \cdot \sum M_A = \hat{y}_r \cdot \dot{H}_A$$

$$\sum M_A = M_{rd}^R + r_{CP/A}^R \times F_{aero}^R + r_{R/G}^R \times m_R g + M_k^R$$

$$\dot{H}_A = I^R \dot{\omega}_R + \omega_R \times I^R \omega_R + r_{R/A}^R \times m_R a_R$$

$$r_{CP/A} = y_{CP} \hat{y}_r + z_{CP} (\alpha) \hat{z}_r$$

Center of Pressure location constant in span-wise direction

Negligible wing mass, but very high angular rate/acceleration

Negligible wing mass, but very high angular rate/acceleration
Rigid Body Dynamics

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mathcal{B})</td>
<td>Body frame</td>
</tr>
<tr>
<td>(\mathcal{R})</td>
<td>Right wing frame</td>
</tr>
<tr>
<td>(\mathcal{L})</td>
<td>Left wing frame</td>
</tr>
<tr>
<td>(CP_R)</td>
<td>Center of pressure of (\mathcal{R})</td>
</tr>
<tr>
<td>(CP_L)</td>
<td>Center of pressure of (\mathcal{L})</td>
</tr>
<tr>
<td>(G)</td>
<td>Center of gravity of (\mathcal{B})</td>
</tr>
<tr>
<td>(R)</td>
<td>Center of gravity of (\mathcal{R})</td>
</tr>
<tr>
<td>(L)</td>
<td>Center of gravity of (\mathcal{L})</td>
</tr>
</tbody>
</table>

- \(M_{rd}\): Rotational damping moment
- \(r_{A/B}\): Position of \(A\) w.r.t. \(B\)
- \(F_{aero}\): Total aerodynamic force
- \(m\): Mass
- \(g\): Gravity vector
- \(\dot{H}_G^A\): Angular momentum of frame \(A\) about \(G\)
- \(I^A\): Inertia tensor of frame \(A\)
- \(\omega_A\): Angular rate of frame \(A\)
- \(a_R\): Acceleration of point \(R\)
Blade-Element Aerodynamics

- Wing is divided spanwise into rectangular, 2D, rigid differential elements
- Differential force $dF_{aero}$ a function of force coefficient $C_F$, local airspeed $V_{\delta w}$, dynamic pressure $q$, reference area $dS$

\[
dF_{aero} = C_F(\alpha)q dS
\]

\[
q = \frac{1}{2} \rho V_{\delta w} \cdot V_{\delta w}
\]

\[
dS = c(r) dr
\]

\[
V_{\delta w} = V_G + V_{A/G} + V_{\delta w/A}
\]
Blade-Element Aerodynamics

- Integrate along wingspan to obtain total force $F_{aero}$

$$dF_{aero} = C_F(\alpha)q dS$$

$$F_{aero} = \frac{1}{2} C_F(\alpha) \rho \int_0^R \mathbf{V}_{\delta w} \cdot \mathbf{V}_{\delta w} c(r) dr$$

- Angle of attack $\alpha$ approximately constant along wingspan, because velocity $\mathbf{V}_{\delta w}$ is dominated by angular rate $\omega_R$

$$\alpha(t) = \tan^{-1} \frac{\mathbf{V}_{\delta w} \cdot \mathbf{\hat{x}}_w}{\mathbf{V}_{\delta w} \cdot \mathbf{\hat{z}}_w}$$

$$\mathbf{V}_{\delta w} = \mathbf{V}_G + \mathbf{V}_{A/G} + \mathbf{V}_{\delta w/A}$$

- Integral can be decomposed so that it does not have to be evaluated at each step of simulation
# Blade-Element Aerodynamics

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>Dynamic Pressure</td>
<td>$\rho$</td>
<td>Ambient air pressure</td>
</tr>
<tr>
<td>$V_{\delta w}$</td>
<td>Velocity of differential element</td>
<td>$V_G$</td>
<td>Velocity of robot body CG</td>
</tr>
<tr>
<td>$V_{A/G}$</td>
<td>Velocity of hinge point relative to robot body CG</td>
<td>$V_{\delta w/A}$</td>
<td>Velocity of differential element relative to hinge point</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle of attack</td>
<td>$C_F(\alpha)$</td>
<td>Force coefficient</td>
</tr>
<tr>
<td>$dS$</td>
<td>Differential reference area</td>
<td>$R$</td>
<td>Wingspan</td>
</tr>
<tr>
<td>$r$</td>
<td>Wingspan coordinate</td>
<td>$c(r)$</td>
<td>Chord length</td>
</tr>
</tbody>
</table>

![Diagram of blade-element aerodynamics](image)
Controller Modeling
Controller Modeling Motivation

- Open-loop flight deviates quickly from hovering
- To validate model against hovering flight requires duplicating flight test controller for closed-loop simulations

Video Credit: [Ma K.Y., ’13]
Controller Overview

- Flight test controller detailed in [Ma, K.Y. ‘13]
- Control design replicated in simulation for purpose of validation

Altitude: (PID) Desired lift force
Lateral: (PID) Desired body orientation
Attitude: (PID) Desired torque
Signal: Generate signal for piezoelectric actuators
Altitude Controller

\[ f_{L,des} = -k_{pa} e - k_{ia} \int_0^t e \, d\tau - k_{da} \dot{e} \]

\[ e = z_{des} - z \]

Compute desired lift \( f_{L,des} \) from the error in altitude

- \( f_{L,des} \): Desired lift force
- \( k_{pa} \): Proportional gain
- \( e \): Error
- \( k_{ia} \): Integral gain
- \( k_{da} \): Derivative gain
- \( z_{des} \): Desired altitude
- \( z \): Current altitude
Compute desired body orientation from the position error and velocity error

\[ \hat{z}_{des} = -k_{pl}(r - r_d) - k_{dl}(\dot{r} - \dot{r}_d) \]

\( \hat{z}_{des} \) Desired body vector
\( k_{pl} \) Proportional gain
\( r \) Position of robot
\( r_d \) Desired position of robot
\( k_{dl} \) Derivative gain
Attitude Controller

\[ \tau_{\text{des}} = -k_p \hat{z}_{\text{des}} - k_d L \chi \]

where the body Euler angles \( \Theta \) are used to compute

\[ \begin{align*}
\omega &= L \dot{\Theta} \\
\chi &= \frac{S}{S + \lambda} \Theta
\end{align*} \]

\( \tau_{\text{des}} \) Desired body torque
\( k_p \) Proportional gain
\( \hat{z}_{\text{des}} \) Desired body vector
\( k_d \) Derivative gain
\( L \) Nonorthogonal transformation matrix
\( \omega \) Body angular rate
Model Validation
Forced Response

High frequency forced response of simulation closely matches experimental data
Open-loop Flight

Simulation (bottom) shows similar instability to experiment (top) in open-loop flight
Closed-loop Flight

Qualitative comparison of simulated (left) experimental (right) closed-loop trajectories
Simulation

Closed-loop simulation of hovering flight
Flight Comparisons

Video Credit: (top left and top right) [Ma K.Y., ’13]
Future Work and Conclusion

- **Proportional Integral Filter (PIF) Compensator**
  - Submitted to CDC ’17
  - Based on linearization of full equations of motion about hovering

- **Intelligent control**
  - Preliminary work: [Clawson, T.S. ’16]
  - Use adaptive control architecture to learn on-line

- **Detailed dynamics analysis**
  - Analyze periodic maneuvers and find set points
  - Determine stability of various set points

This model combines accurate aerodynamic force calculations with dynamic modeling to create an integrative flight model
A Blade Element Approach to Modeling Aerodynamic Flight of an Insect-scale Robot

Taylor S. Clawson, Sawyer B. Fuller¹, Robert J. Wood², Silvia Ferrari

¹Mechanical Engineering, University of Washington, Seattle, WA
²Engineering and Applied Sciences + Wyss Institute, Harvard University, Cambridge, MA

Further questions: Taylor Clawson
tsc83@cornell.edu

Related Work