LISC

LABORATORY FOR INTELLIGENT
SYSTEMS AND CONTROLS

A Blade Element Approach to Modeling Aerodynamic Flight of an Insect-scale Robot

CORNELL UNIVERSITY
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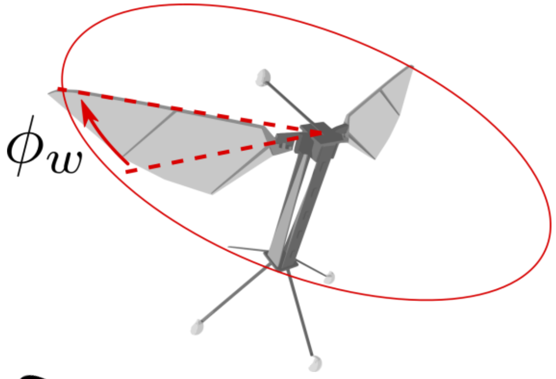
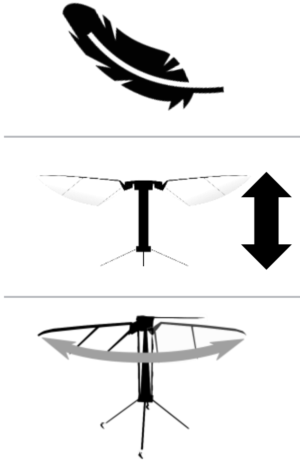
Introduction

RoboBee Background

- Wing stroke angle ϕ_w controlled independently for each wing
- Thrust and body torques controlled by modulating stroke angle commands





100 mg

14 mm

120 Hz

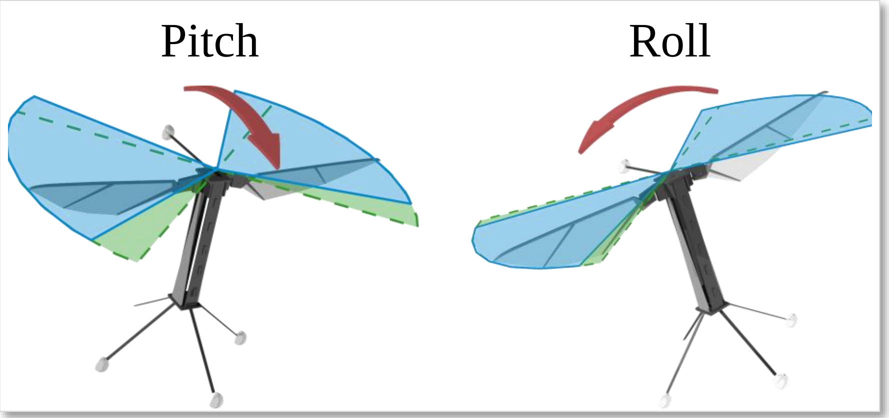


Image Credit: [Ma K.Y., ’12], [Ma K.Y., ’13]

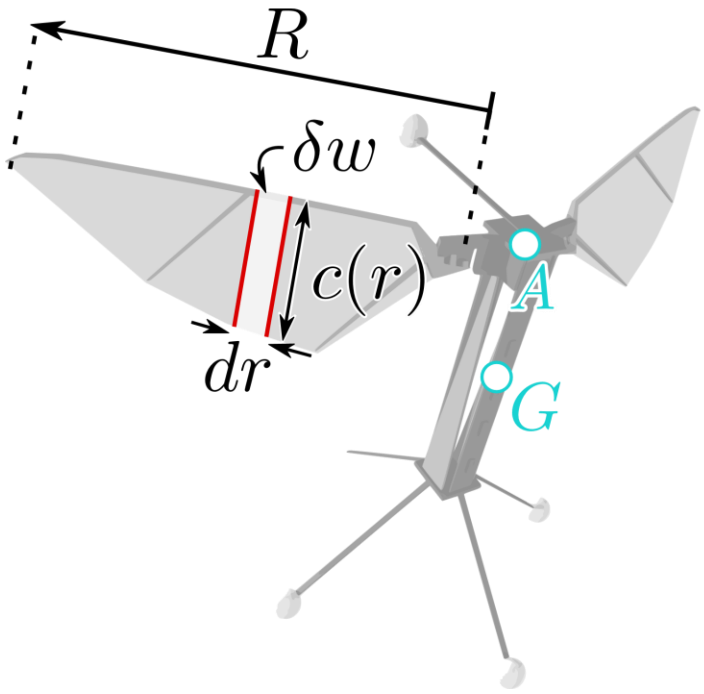
Video of RoboBee test flight courtesy of the Harvard Microrobotics Lab

Introduction and Motivation

- Applications
  - Navigation in cluttered environments, requiring precise reference tracking
  - Robust stabilization, subject to large disturbances such as winds and gusts
- Research Goals
  - Control design, implementation, and guarantees
  - Develop high-fidelity simulation tools
- Previous work
  - Simplified RoboBee Flight Model [Fuller, S.B. ’14], [Chirarattananon, P. ’16]
    - 6 DOF body motion, no wing modeling
    - Linearized, uncoupled, stroke-averaged aerodynamic forces
    - Controlled with hierarchical PID and iterative learning
  - RoboBee Wing Aerodynamics [Whitney, J.P. ’10], [Jafferis, N.T. ’16]
    - Model wing aerodynamics with blade-element theory
    - Omit body dynamics (constant body position and orientation)

Blade-Element Overview

- Wing is divided span-wise into rigid 2D differential elements
- Differential forces are computed for each element, and then integrated along wingspan for total force on wing
- Tuned to provide close approximation of actual forces in an expression that is:
  - Closed-form
  - Computationally-efficient
  - Provides insight into dominant underlying physics



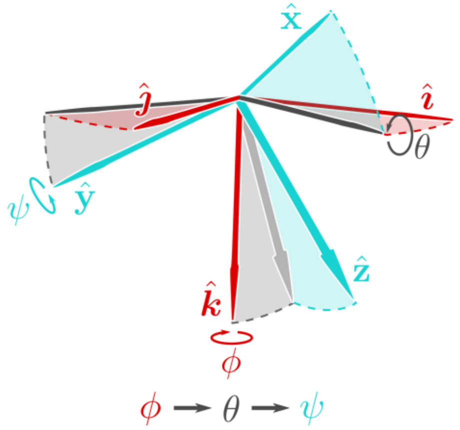
Model Description

Modeling Setup

- 3 Rigid bodies
  - Main body + two wings
- 8 DOF model
  - Main body: 6 DOF
  - Wings: 1 DOF each (pitch angle ψ_w)
    - Stroke angle ϕ_w treated as an input
    - No stroke-plane deviation θ_w

Fixed frame to body frame

$$\mathcal{F}\left\{\hat{\boldsymbol{\imath}},\hat{\boldsymbol{\jmath}},\hat{\boldsymbol{k}}\right\} \Rightarrow \mathcal{B}\{\hat{\boldsymbol{x}},\hat{\boldsymbol{y}},\hat{\boldsymbol{z}}\}$$

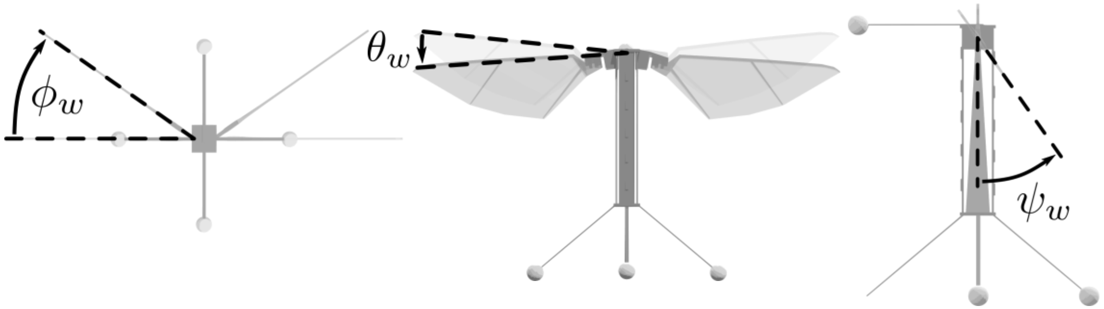


Similar for:

- $\mathcal{B}$ to left wing $\mathcal{L}\{\hat{\boldsymbol{x}}_l,\hat{\boldsymbol{y}}_l,\hat{\boldsymbol{z}}_l\}$
- $\mathcal{B}$ to right wing $\mathcal{R}\{\hat{\boldsymbol{x}}_r,\hat{\boldsymbol{y}}_r,\hat{\boldsymbol{z}}_r\}$

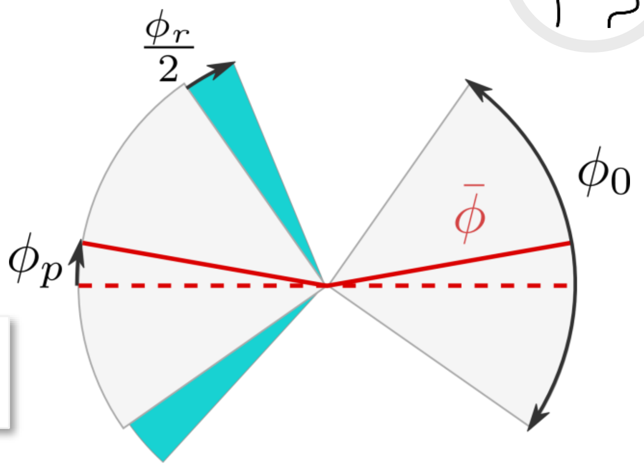
Wing Euler Angles

Stroke Angle	Stroke-Plane Deviation	Wing Pitch



States and Inputs

$$\boldsymbol{x} = [\boldsymbol{\Theta}^T \quad \boldsymbol{r}^T \quad \boldsymbol{\Theta}_r^T \quad \boldsymbol{\Theta}_l^T \quad \dot{\boldsymbol{\Theta}}^T \quad \dot{\boldsymbol{r}}^T \quad \dot{\boldsymbol{\Theta}}_r^T \quad \dot{\boldsymbol{\Theta}}_l^T]^T$$
$$\boldsymbol{u} = [\phi_0 \quad \phi_p \quad \phi_r]^T$$



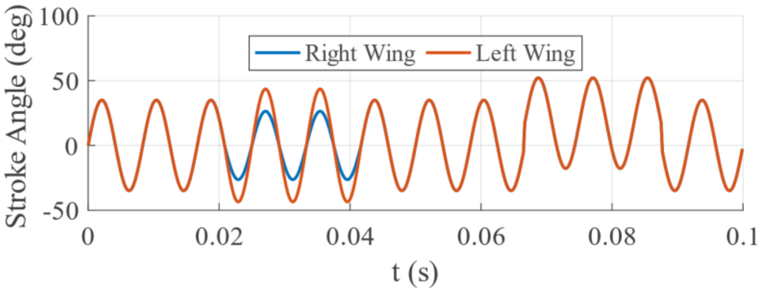
Stroke angle trajectory ϕ_w modeled as a function of input $\boldsymbol{u}$ following linear second-order system:

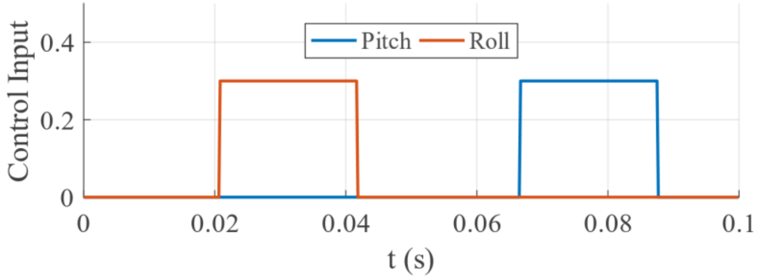
$$\ddot{\phi}_w(t) + 2\zeta\omega_n\dot{\phi}_w(t) + \omega_n^2\phi_w(t) = A_w \sin(\omega_f t) + \bar{\phi}_w$$

For the right wing, for example,

$$A_w = \phi_0 - \frac{\phi_r}{2}, \quad \bar{\phi}_w = -\phi_p$$

$\boldsymbol{x}$	State	ϕ_W	Wing stroke angle
$\boldsymbol{u}$	Control Input	ϕ_0	Nominal stroke amplitude
$\boldsymbol{\Theta}$	Body orientation	ϕ_p	Pitch input
$\boldsymbol{r}$	Body position	ϕ_r	Roll input
$\boldsymbol{\Theta}_r$	Right wing orientation	A_w	Wing stroke amplitude
ω_f	Flapping frequency	$\bar{\phi}_w$	Mean stroke angle





Rigid Body Dynamics

- Angular momentum balance about body CG:

$$\sum \boldsymbol{M}_G = \sum \dot{\boldsymbol{H}}_G$$
$$\sum \boldsymbol{M}_G^{\mathcal{L}} + \sum \boldsymbol{M}_G^{\mathcal{R}} = \dot{\boldsymbol{H}}_G^{\mathcal{B}} + \dot{\boldsymbol{H}}_G^{\mathcal{L}} + \dot{\boldsymbol{H}}_G^{\mathcal{R}}$$

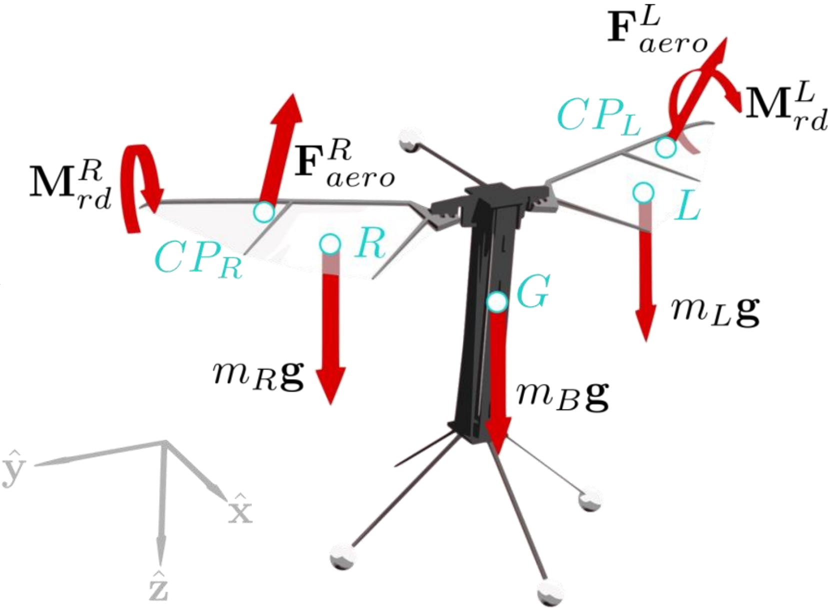
- Blade-element theory used to calculate aerodynamic forces and moments

- Aerodynamic forces act at instantaneous centers of pressure CP_L, CP_R

$$\sum \boldsymbol{M}_G^{\mathcal{L}} = \boldsymbol{M}_{rd}^{\mathcal{L}} + \boldsymbol{r}_{CP_L/G} \times \boldsymbol{F}_{aero}^{\mathcal{L}} + \boldsymbol{r}_{L/G} \times m_{\mathcal{L}} \boldsymbol{g}$$

- Angular momentum about G calculated as a sum of contributions from each frame

$$\dot{\boldsymbol{H}}_G^{\mathcal{B}} = \boldsymbol{I}^{\mathcal{B}} \dot{\boldsymbol{\omega}}_{\mathcal{B}} + \boldsymbol{\omega}_{\mathcal{B}} \times \boldsymbol{I}^{\mathcal{B}} \boldsymbol{\omega}_{\mathcal{B}}$$
$$\dot{\boldsymbol{H}}_G^{\mathcal{L}} = \boldsymbol{I}^{\mathcal{L}} \dot{\boldsymbol{\omega}}_{\mathcal{L}} + \boldsymbol{\omega}_{\mathcal{L}} \times \boldsymbol{I}^{\mathcal{L}} \boldsymbol{\omega}_{\mathcal{L}} + \boldsymbol{r}_{L/G} \times m_{\mathcal{L}} \boldsymbol{a}_L$$



Wing Rigid Body Dynamics

- Single DOF: wing pitch ψ_w
  - Angular momentum balance in span-wise direction

$$\hat{\boldsymbol{y}}_r \cdot \sum \boldsymbol{M}_A = \hat{\boldsymbol{y}}_r \cdot \dot{\boldsymbol{H}}_A$$

$$\sum \boldsymbol{M}_A = \boldsymbol{M}_{rd}^{\mathcal{R}} + \underline{\boldsymbol{r}_{CP_R/A}} \times \boldsymbol{F}_{aero}^{\mathcal{R}} + \overbrace{\boldsymbol{r}_{R/G} \times m_{\mathcal{R}} \boldsymbol{g}}^{\text{Negligible}} + \boldsymbol{M}_k^{\mathcal{R}}$$

$$\dot{\boldsymbol{H}}_A = \boldsymbol{I}^{\mathcal{R}} \dot{\boldsymbol{\omega}}_{\mathcal{R}} + \boldsymbol{\omega}_{\mathcal{R}} \times \boldsymbol{I}^{\mathcal{R}} \boldsymbol{\omega}_{\mathcal{R}} + \underbrace{\boldsymbol{r}_{R/A} \times m_{\mathcal{R}} \underline{\boldsymbol{a}_R}}_{\text{Non-Negligible}}$$

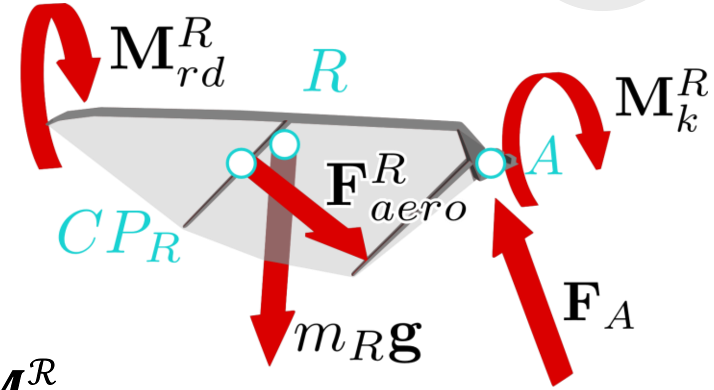
$$\boxed{\boldsymbol{r}_{CP_R/A} = y_{CP} \hat{\boldsymbol{y}}_r + z_{CP}(\alpha) \hat{\boldsymbol{z}}_r}$$

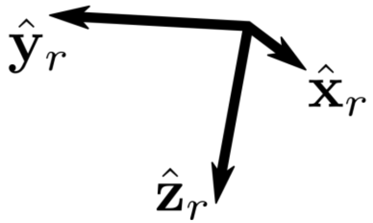
Center of Pressure location constant in span-wise direction

$$\boxed{\boldsymbol{a}_R = \boldsymbol{a}_G + \boldsymbol{a}_{A/G} + \underline{\boldsymbol{a}_{R/A}}}$$

$$\boxed{\boldsymbol{a}_{R/A} = \dot{\boldsymbol{\omega}}_{\mathcal{R}} \times \boldsymbol{r}_{R/A} + \boldsymbol{\omega}_{\mathcal{R}} \times (\boldsymbol{\omega}_{\mathcal{R}} \times \boldsymbol{r}_{R/A})}$$

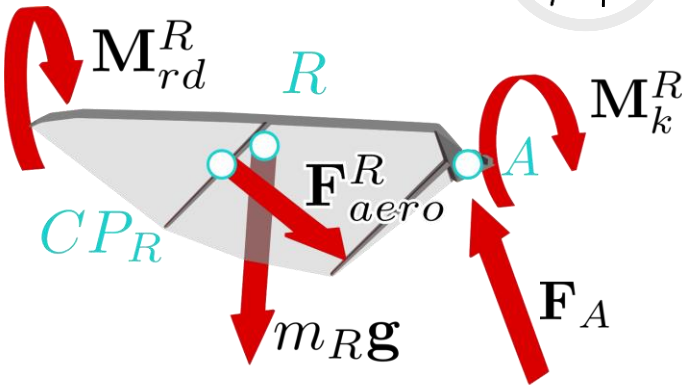
Negligible wing mass, but very high angular rate/acceleration



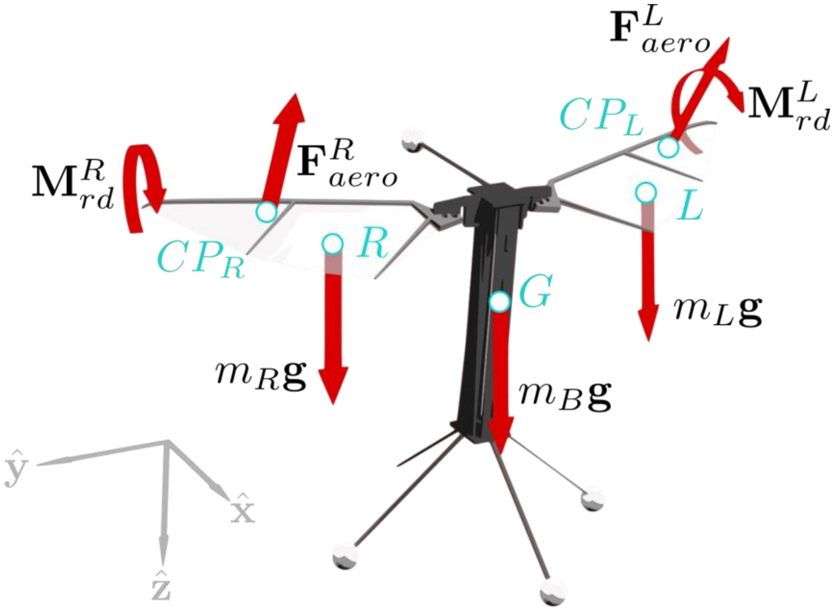


Rigid Body Dynamics

$\mathcal{B}$	Body frame	CP_L	Center of pressure of $\mathcal{L}$
$\mathcal{R}$	Right wing frame	G	Center of gravity of $\mathcal{B}$
$\mathcal{L}$	Left wing frame	R	Center of gravity of $\mathcal{R}$
CP_R	Center of pressure of $\mathcal{R}$	L	Center of gravity of $\mathcal{L}$

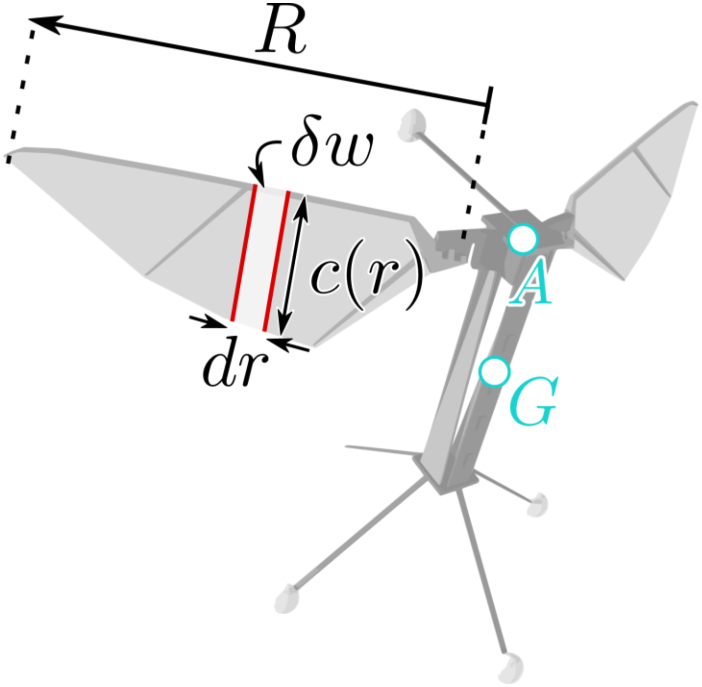


$\boldsymbol{M}_{rd}$	Rotational damping moment
$\boldsymbol{r}_{A/B}$	Position of A w.r.t. B
$\boldsymbol{F}_{aero}$	Total aerodynamic force
m	Mass
$\boldsymbol{g}$	Gravity vector
$\dot{\boldsymbol{H}}_{G}^{\mathcal{A}}$	Angular momentum of frame $\mathcal{A}$ about G
$\boldsymbol{I}^{\mathcal{A}}$	Inertia tensor of frame $\mathcal{A}$
$\boldsymbol{\omega}_{\mathcal{A}}$	Angular rate of frame $\mathcal{A}$
$\boldsymbol{a}_R$	Acceleration of point R



Blade-Element Aerodynamics

- Wing is divided spanwise into rectangular, 2D, rigid differential elements
- Differential force dF_{aero} a function of force coefficient C_F, local airspeed $\boldsymbol{V}_{\delta w}$, dynamic pressure q, reference area dS



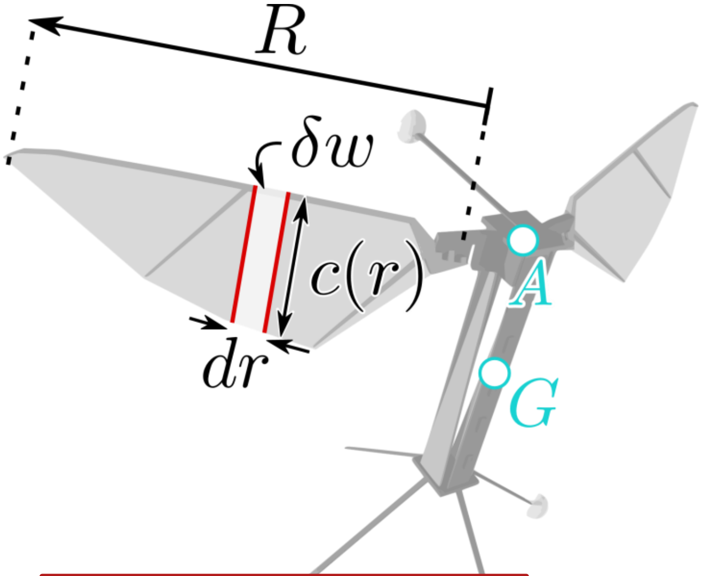
$$\begin{aligned} dF_{aero} &= C_F(\alpha) q dS \\ q &= \frac{1}{2}\rho \boldsymbol{V}_{\delta w} \cdot \boldsymbol{V}_{\delta w} \\ dS &= c(r)dr \\ \boldsymbol{V}_{\delta w} &= \boldsymbol{V}_G + \boldsymbol{V}_{A/G} + \boldsymbol{V}_{\delta w/A} \end{aligned}$$

Blade-Element Aerodynamics

- Integrate along wingspan to obtain total force F_{aero}

$$dF_{aero} = C_F(\alpha)qdS$$

$$\boxed{F_{aero} = \frac{1}{2}C_F(\alpha)\rho\int_0^R \boldsymbol{V}_{\delta w}\cdot\boldsymbol{V}_{\delta w}\,c(r)dr}$$



- Angle of attack α approximately constant along wingspan, because velocity $\boldsymbol{V}_{\delta w}$ is dominated by angular rate $\boldsymbol{\omega}_R$

$$\alpha(t) = \tan^{-1}\frac{\boldsymbol{V}_{\delta w}\cdot\hat{\boldsymbol{x}}_w}{\boldsymbol{V}_{\delta w}\cdot\hat{\boldsymbol{z}}_w}$$

small

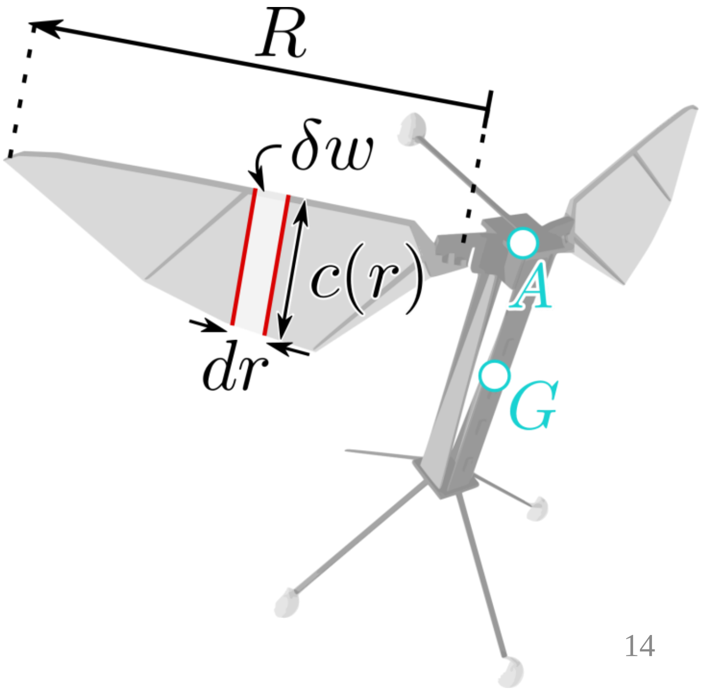
$$\boldsymbol{V}_{\delta w} = \underline{\boldsymbol{V}_G} + \underline{\boldsymbol{V}_{A/G}} + \underline{\boldsymbol{V}_{\delta w/A}}$$

$$\boxed{\boldsymbol{V}_{\delta w/A} = \boldsymbol{\omega}_{\mathcal{R}} \times \boldsymbol{r}_{\delta w/A}}$$

- Integral can be decomposed so that it does not have to be evaluated at each step of simulation

Blade-Element Aerodynamics

q	Dynamic Pressure	ρ	Ambient air pressure
$\boldsymbol{V}_{\delta w}$	Velocity of differential element	$\boldsymbol{V}_G$	Velocity of robot body CG
$\boldsymbol{V}_{A/G}$	Velocity of hinge point relative to robot body CG	$\boldsymbol{V}_{\delta w/A}$	Velocity of differential element relative to hinge point
α	Angle of attack	$C_F(\alpha)$	Force coefficient
dS	Differential reference area	R	Wingspan
r	Wingspan coordinate	$c(r)$	Chord length



Controller Modeling

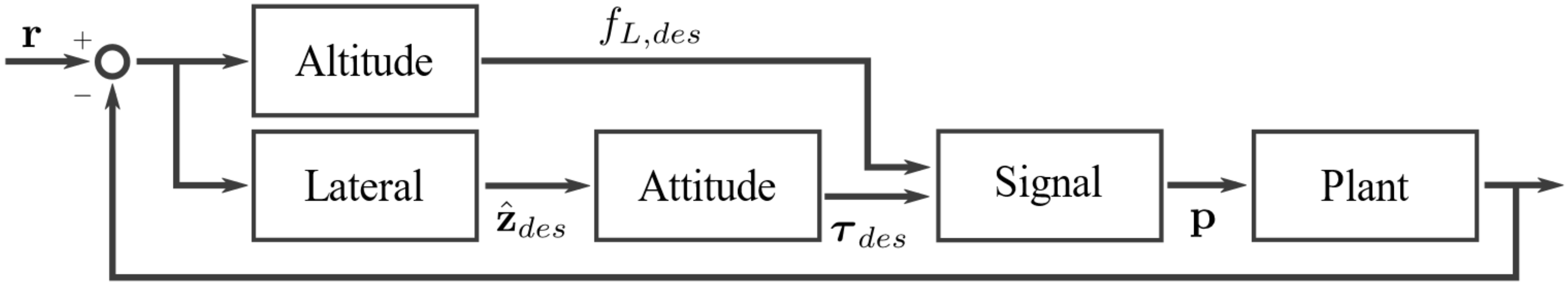
Controller Modeling Motivation

- Open-loop flight deviates quickly from hovering
- To validate model against hovering flight requires duplicating flight test controller for closed-loop simulations

Video Credit: [Ma K.Y., ’13]

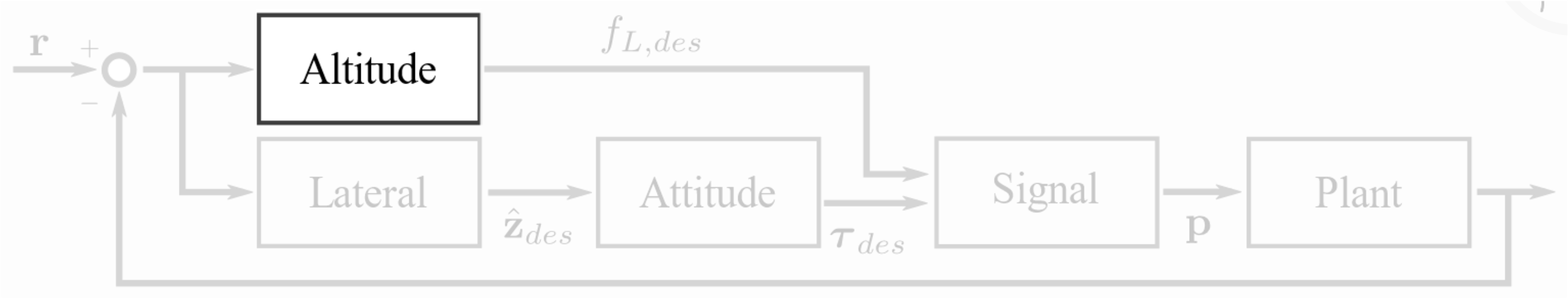
Controller Overview

- Flight test controller detailed in [Ma, K.Y. ‘13]
- Control design replicated in simulation for purpose of validation



Altitude:	(PID) Desired lift force
Lateral:	(PID) Desired body orientation
Attitude:	(PID) Desired torque
Signal:	Generate signal for piezoelectric actuators

Altitude Controller



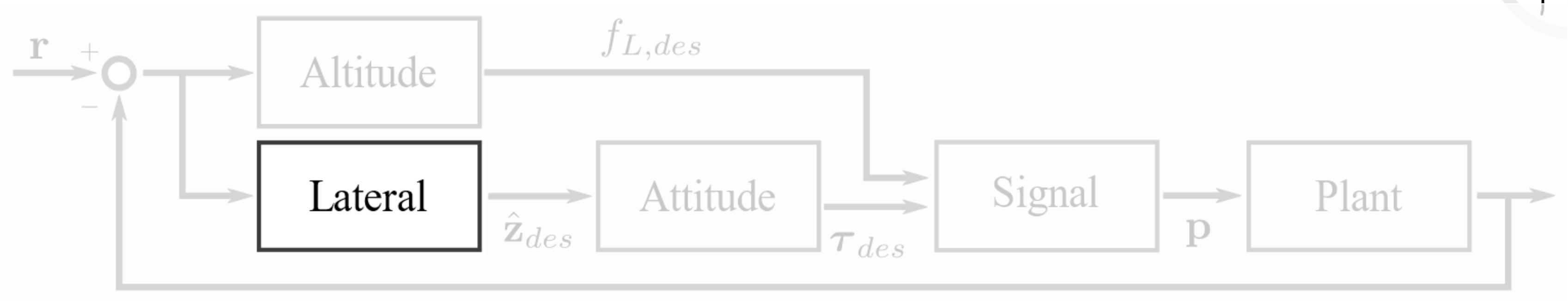
$$f_{L,des} = -k_{pa}e - k_{ia}\int_0^t e\, d\tau - k_{da}\dot{e}$$

$$e = z_{des} - z$$

Compute desired lift $f_{L,des}$ from the error in altitude

$f_{L,des}$	Desired lift force
k_{pa}	Proportional gain
e	Error
k_{ia}	Integral gain
k_{da}	Derivative gain
z_{des}	Desired altitude
z	Current altitude

Lateral Controller

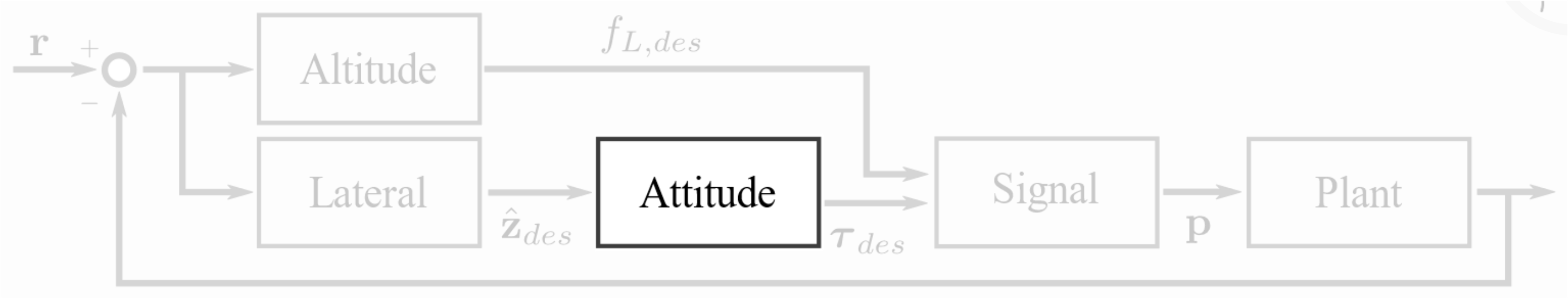


$$\hat{\mathbf{z}}_{des} = -k_{pl}(\mathbf{r} - \mathbf{r}_d) - k_{dl}(\dot{\mathbf{r}} - \dot{\mathbf{r}}_d)$$

Compute desired body orientation from the position error and velocity error

$\hat{\mathbf{z}}_{des}$	Desired body vector
k_{pl}	Proportional gain
$\mathbf{r}$	Position of robot
$\mathbf{r}_d$	Desired position of robot
k_{dl}	Derivative gain

Attitude Controller



$$\boldsymbol{\tau}_{des} = -k_p \hat{\mathbf{z}}_{des} - k_d L \boldsymbol{\chi}$$

where the body Euler angles $\boldsymbol{\Theta}$ are used to compute

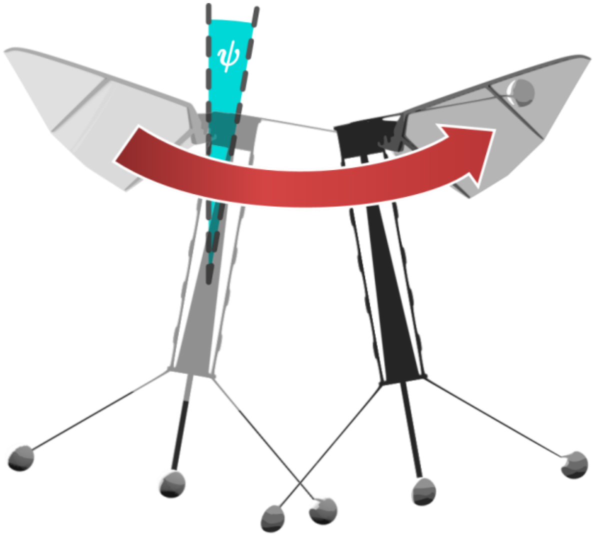
$$\boldsymbol{\omega} = L\dot{\boldsymbol{\Theta}}$$

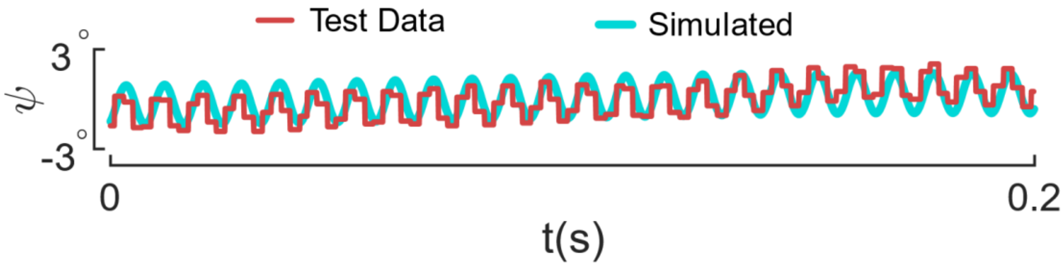
$$\boldsymbol{\chi} = \frac{s}{s+\lambda}\boldsymbol{\Theta}$$

$\boldsymbol{\tau}_{des}$	Desired body torque
k_p	Proportional gain
$\hat{\mathbf{z}}_{des}$	Desired body vector
k_d	Derivative gain
L	Nonorthogonal transformation matrix
$\boldsymbol{\omega}$	Body angular rate

Model Validation

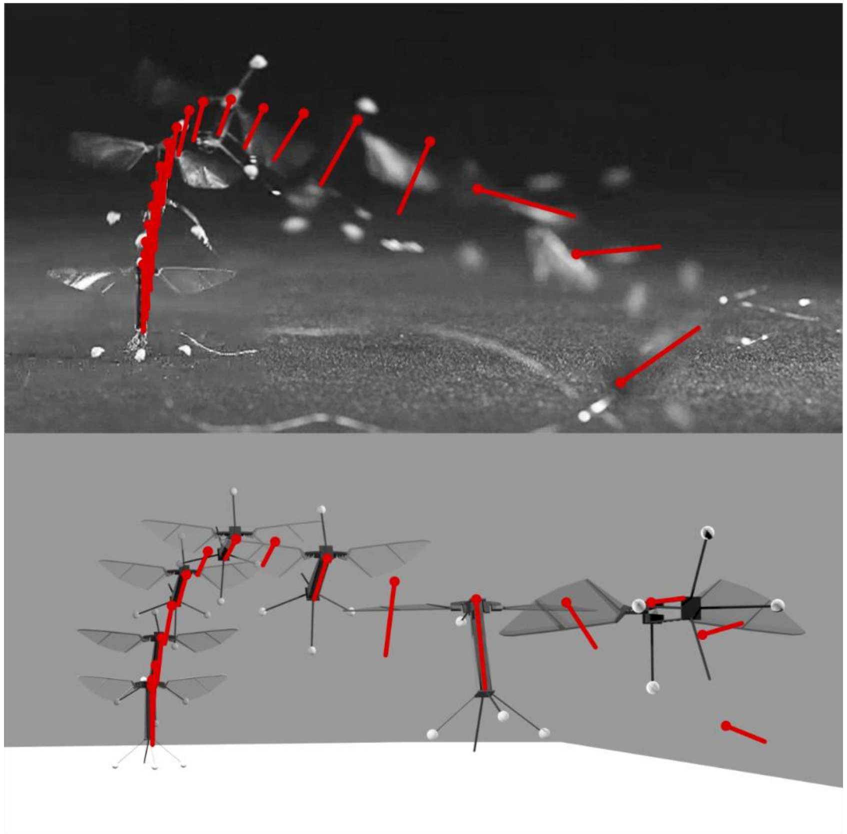
Forced Response





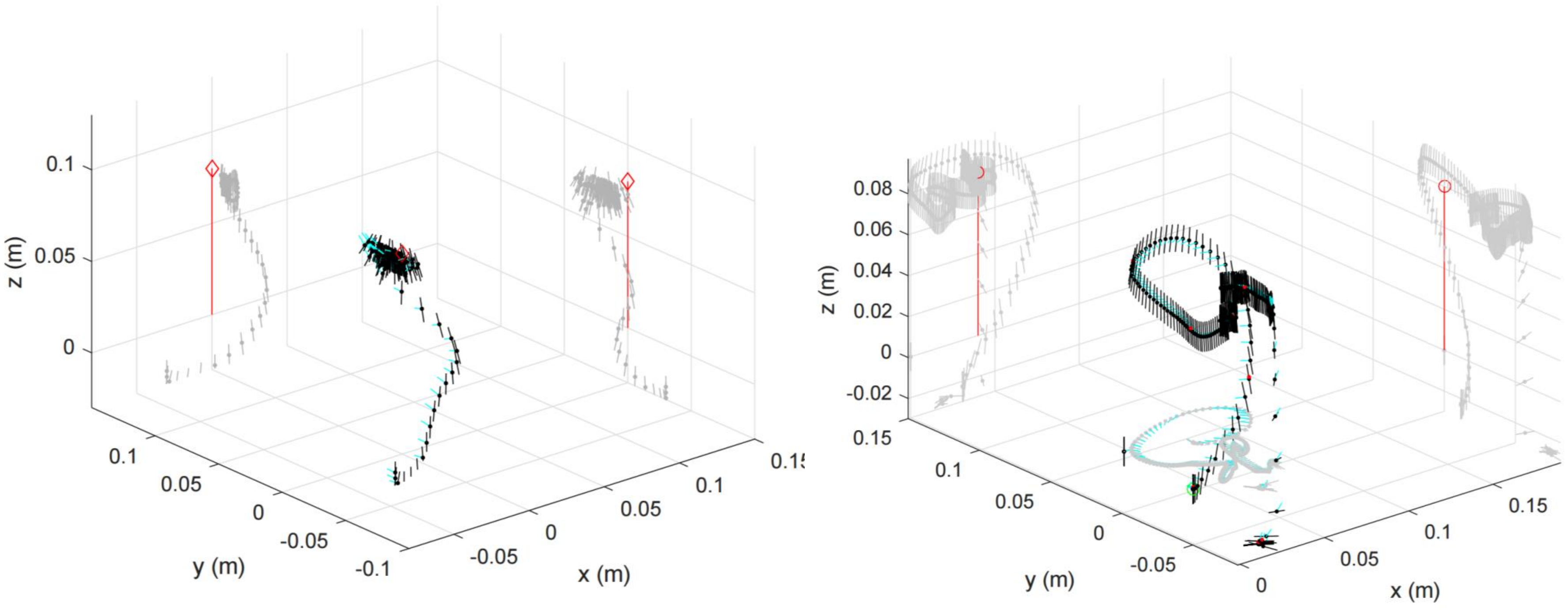
High frequency forced response of simulation closely matches experimental data

Open-loop Flight



Simulation (bottom) shows similar instability to experiment (top) in open-loop flight

Closed-loop Flight

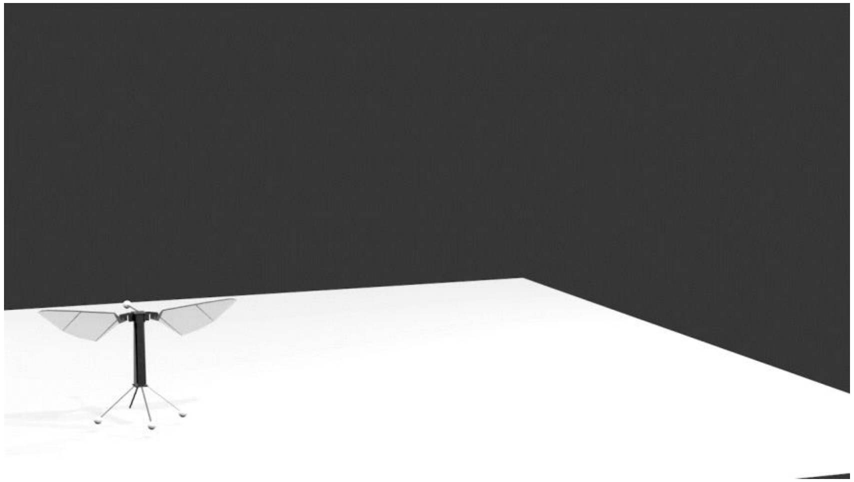


Qualitative comparison of simulated (left) experimental (right) closed-loop trajectories

Simulation

Closed-loop simulation of hovering flight

Flight Comparisons



Video Credit: (top left and top right) [Ma K.Y., ’13]

Future Work and Conclusion

- Proportional Integral Filter (PIF) Compensator
  - Submitted to CDC ’17
  - Based on linearization of full equations of motion about hovering
- Intelligent control
  - Preliminary work: [Clawson, T.S. ’16]
  - Use adaptive control architecture to learn on-line
- Detailed dynamics analysis
  - Analyze periodic maneuvers and find set points
  - Determine stability of various set points

This model combines accurate aerodynamic force calculations with dynamic modeling to create an integrative flight model

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Further questions: Taylor Clawson
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Related Work

Clawson, Taylor S., et al. "Spiking neural network (SNN) control of a flapping insect-scale robot." *Decision and Control (CDC), 2016 IEEE 55th Conference on*. IEEE, 2016.