Multiscale Adaptive Sensor Systems

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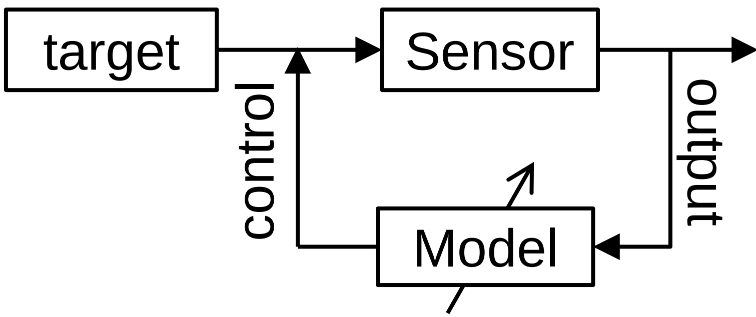
LISC
LABORATORY FOR INTELLIGENT SYSTEMS AND CONTROLS

Introduction and Motivation

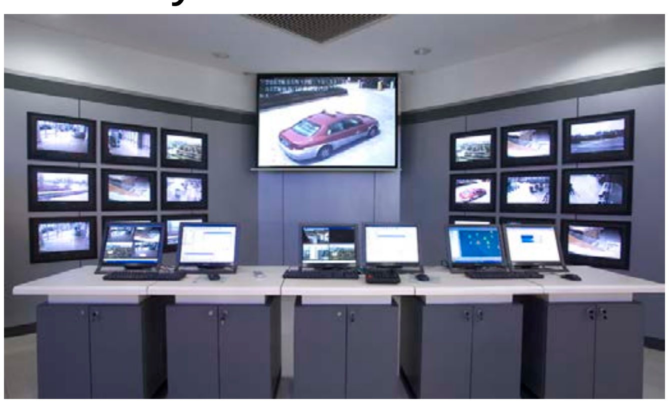
- Sensor planning: control reconfigurable sensors for collaborative gather information in contested communication environments



- Tracking endangered species

[1]

- Security surveillance task



[3]

- Search and Rescue

[2]

[1] http://www.ibtimes.co.uk/africa-safari-drones-remote-controlled-buggies-capture-intimate-wildlife-shots-photos-1524742
[2] http://dowley.com/Services/VideoSurveillance/tabid/91/Default.aspx
[3] http://cadet63.rssing.com/chan-9332332/all_p12.html

Literature Review

- Information-driven sensor planning
  - Information value comparison [Polcari 13, Kastella 97, ...]
  - Cell decomposition [Cai 09, Paull 10, Ferrari 09, 12, ...]
  - Probability road maps and trees [Zhang 09, Lu 10, 12, 14, ...]
  - Graphical model [Krause 06, 10, 12, Singh 09, Guestrin 05, Meliou 07, Srinivas 12, Le Ny 09, ...]
  - Advantage:
    a. Represent information value of measurements for improving the target model
    b. Can be calculated before measurements are obtained
  - Disadvantage:
    a. Can only be applied when target models are known
    b. Assumptions too restricted: stationary target; discrete and finite control space; unbounded sensor field of view; unconstrained sensor dynamics, ...

Problem Formulation

Assumptions

- Workspace: $\mathcal{W} \subset \mathbb{R}^d$, where $d \in \mathbb{Z}^+$
- Target kinematics: unknown form

$$\dot{\mathbf{x}}_j(t) = \mathbf{f}_j[\mathbf{x}_j(t)] \triangleq \mathbf{v}_j(t), \quad j = 1, \ldots, N$$

- Sensor dynamics: constrained

$$\mathbf{s}(k+1) = \mathbf{A}\mathbf{s}(k) + \mathbf{B}\mathbf{u}(k)$$
$$\mathbf{s}(k) \in \mathcal{A}, \ \mathbf{u}(k) \in \mathcal{U}$$

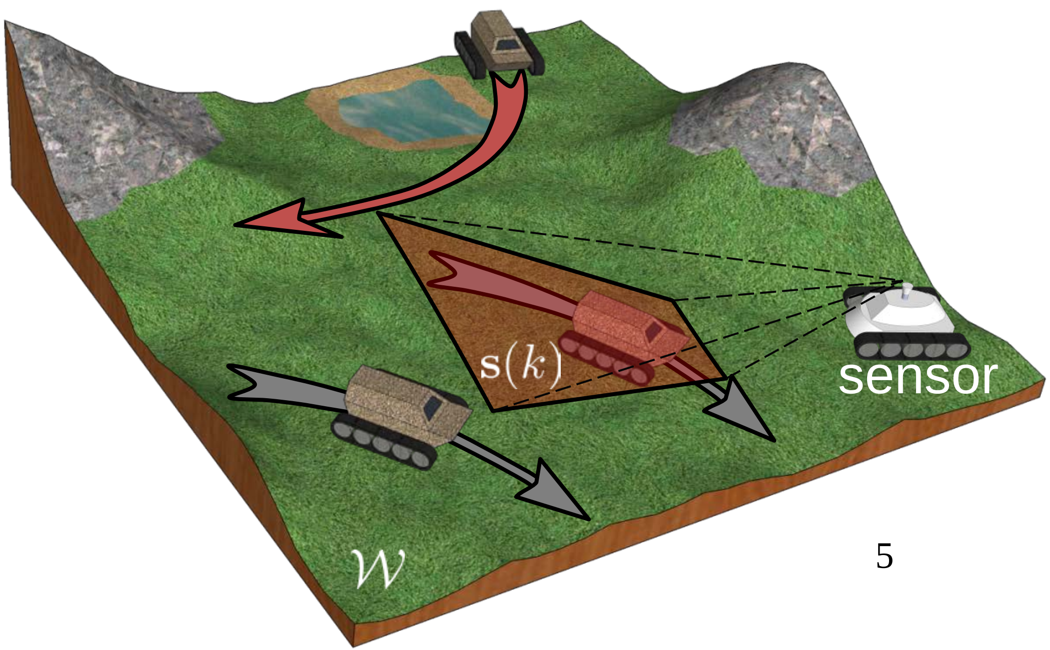
- Sensor field of view: bounded

$$\mathcal{S}[\mathbf{s}(k)] = \mathcal{S}(k) \subset \mathcal{W}$$

- Measurement model:

$$\mathbf{m}_j(k) = [\mathbf{x}_j^T(k) \quad \mathbf{v}_j^T(k)]^T + \boldsymbol{\nu}$$

- $\mathbf{x}_j$: target state
- $\mathbf{v}_j$: target velocity
- $\mathbf{s}$: sensor state
- $\mathbf{u}$: sensor control input
- $\mathcal{A}$: admissible sensor state
- $\mathcal{U}$: admissible control input
- $\mathcal{S}$: sensor FOV
- $\mathbf{m}_j$: target measurement
- $\boldsymbol{\nu}$: measurement noise



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Problem Formulation

- Objective function: $\mathscr{L}[\mathbf{s}(k), \mathbf{u}(k)|\mathcal{M}(k)]$
  - Information brought about by sensor measurements
  - Nonmyopic: conditioned on all previous measruements, $\mathcal{M}(k)$
- **Sensor Planning**: *Given the reward function,* $\mathscr{L}[\mathbf{s}(k), \mathbf{u}(k)|\mathcal{M}(k)]$, *that evaluates the information value of the sensor state,* $\mathbf{s}(k) \in \mathcal{A}$, *and control vector,* $\mathbf{u}(k) \in \mathcal{U}$, *for learning the target kinematic models, find the optimal control,* $\mathbf{u}^*(k)$, *that maximizes the reward and is subject to the sensor dynamics for the control horizon* K,

$$\begin{aligned} \max_{\mathbf{u}(k)\in\mathcal{U}} \quad & J = \sum_{k=1}^{K} \mathscr{L}[\mathbf{s}(k), \mathbf{u}(k)|\mathcal{M}(k)] \\ \text{s. t.} \quad & \mathbf{s}(k+1) = \mathbf{A}\mathbf{s}(k) + \mathbf{B}\mathbf{u}(k) \\ & \mathbf{s}(0) = \mathbf{s}_0, \quad \mathbf{s}(k) \in \mathcal{A}, \quad \mathbf{u}(k) \in \mathcal{U} \end{aligned}$$

where $\mathbf{s}_0$ *is the initial sensor state.*

Example

Information Sufficiency

Motivation: Develop planning and control algorithms for collaborative networks with intermittent communications

- Existing decentralized optimization methods assume constant communications (network is a connected graph) or detailed prior information (perfect models)
- Consider networks in which some or all nodes (agents) may be disconnected some of the time, and there is no or little prior information (high uncertainty)
- Agents aim to construct probabilistic model from data
- Disconnected agents can determine when their own local information is insufficient, and it is time to reestablish communications

Model a spatial phenomenon:

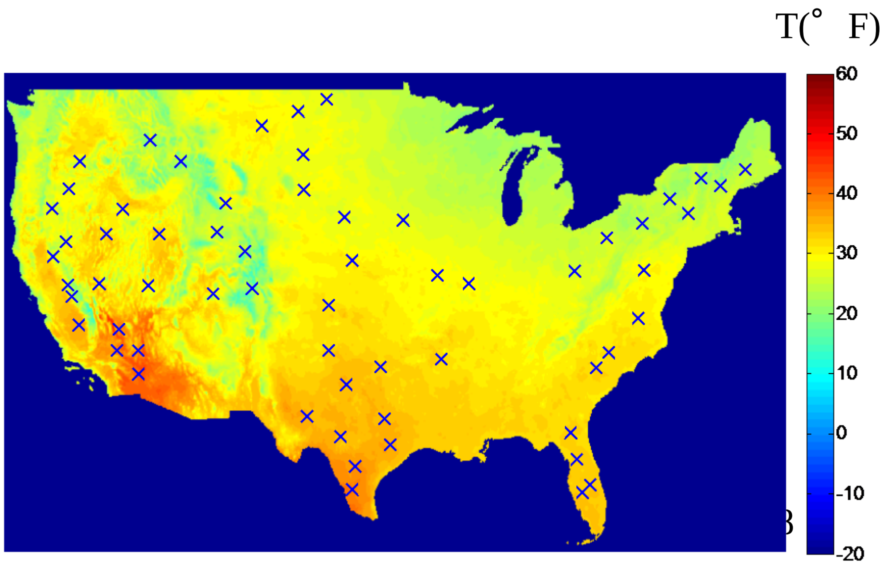
$$g(\mathbf{x}), \mathbf{x} \in \mathcal{A}$$

- Max temperature over a 2D ROI

$$\mathcal{A} \subset \mathbb{R}^2$$

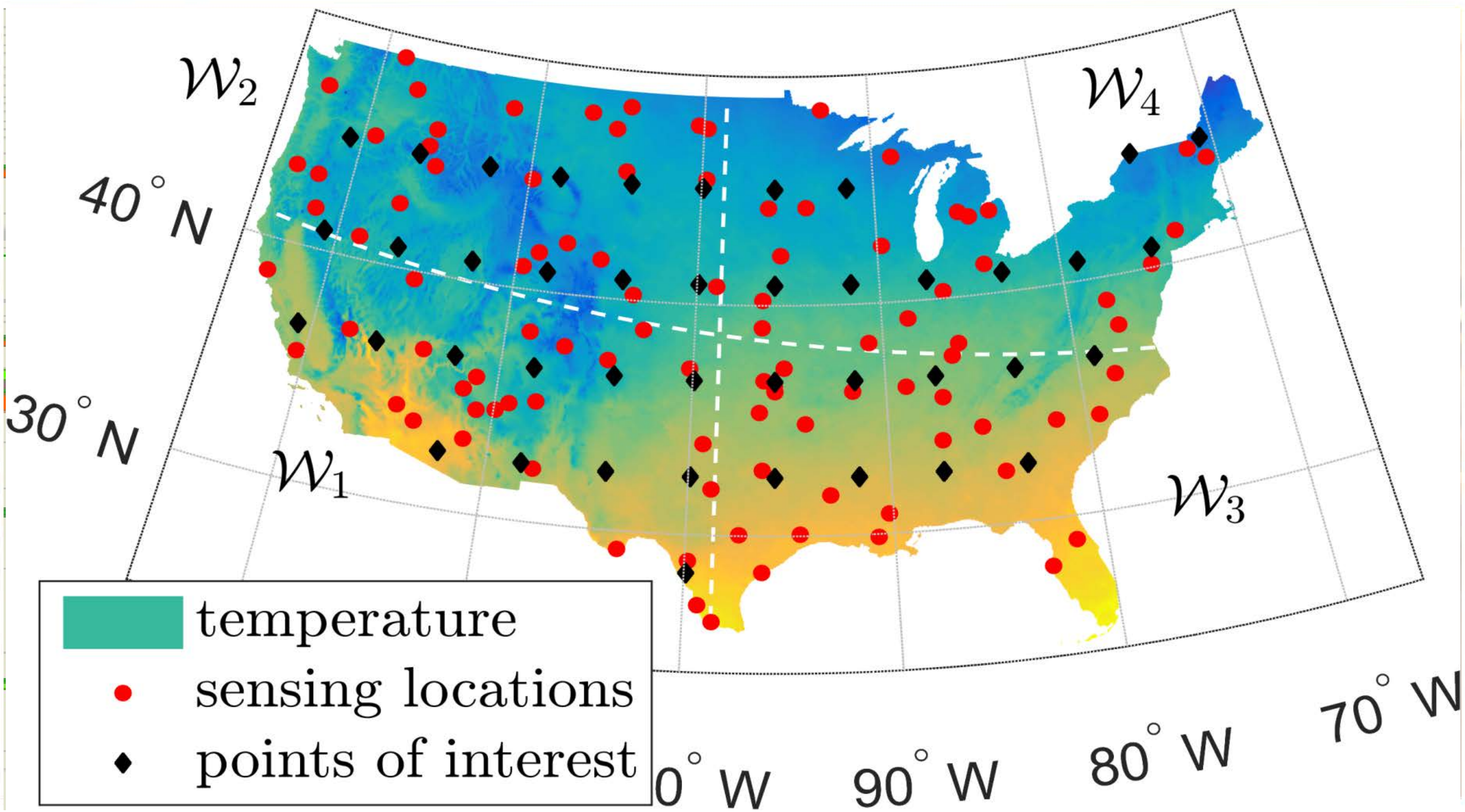
- Time invariant
- Observable at a set of target locations:

$$\mathcal{T} = \{\mathbf{t}_i | i = 1, \cdots, r\}, \mathbf{t}_i \in \mathcal{A}$$



Applications

Robot Navigation	Traffic Planning	Animal Tracking	Env. Monitor



1: http://acl.mit.edu/projects/cbba.html
2: http://www.caliper.com/transcad/applicationmodules.htm
3: http://www.societyofrobots.com/robotforum/index.php?topic=16714.0
4: PRISM Climate Group, Oregon State University, http://prism.oregonstate.edu, created March 2016.

Model and Information Value

Estimation of spatial phenomenon:

- $g(\mathbf{x}) \xleftarrow{\text{estimation}} f(\mathbf{x}), \mathbf{x} \in \mathcal{A}$
- Measurements: $f(\mathbf{x})$ ~ Gaussian process

Gaussian process:

$$f(\mathbf{x}) \sim GP(\mu(\mathbf{x}), \psi(\mathbf{x}_1,\mathbf{x}_2)); \ \mu(\mathbf{x}) = E[f(\mathbf{x})]$$

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = E[(f(\mathbf{x}_1) - \mu(\mathbf{x}_1))(f(\mathbf{x}_2) - \mu(\mathbf{x}_2))]$$

Planning objective: at time k choose locations and measurements $\{\mathbf{y}_k, z_k\}$ to maximize,

$$D(p(\mathbf{f}(\mathbf{X}_T)|\mathbf{Y}_k,\mathbf{Z}_k)||p(\mathbf{f}(\mathbf{X}_T)|\mathbf{Y}_{k-1},\mathbf{Z}_{k-1}))$$

where,

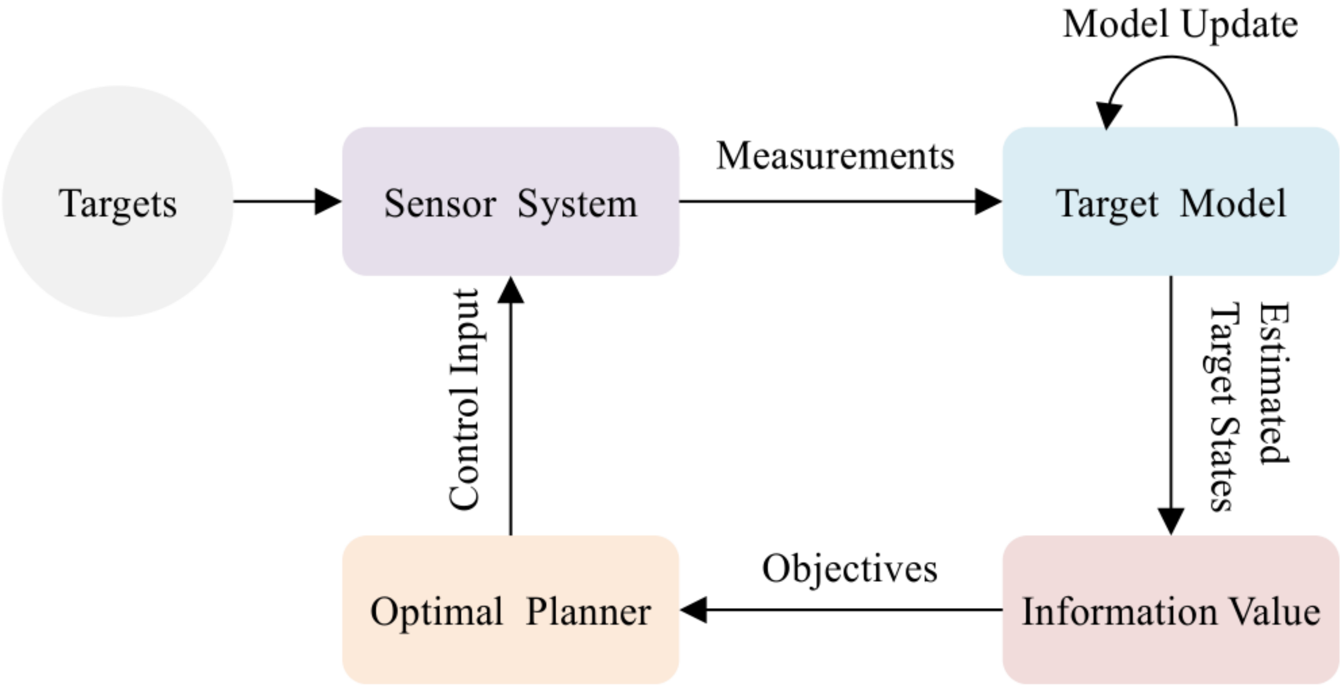
$\mathbf{X}_T = [\mathbf{x}_1 | \ \cdots \ | \mathbf{x}_r], \mathbf{x}_i \in \mathcal{T}, i = 1, \ldots, r$; $\mathbf{Y}_k = [\mathbf{y}_1 | \cdots | \mathbf{y}_k]$; $\mathbf{Z}_k = [z_1 | \cdots | z_k]$

Since $\mathbf{z}_k$ is unknown, optimize **expected discrimination gain (EDG):**

$$\boxed{\begin{aligned}&\hat{\varphi}_D(\mathbf{f}(\mathbf{X}_T); z_k|\mathbf{y}_k, \mathbf{Y}_{k-1}, \mathbf{Z}_{k-1}) = \\ &\int D(p(\mathbf{f}(\mathbf{X}_T)|\mathbf{Y}_{k-1},\mathbf{Z}_{k-1},\mathbf{y}_k,z_k)||p(\mathbf{f}(\mathbf{X}_T)|\mathbf{Y}_{k-1},\mathbf{Z}_{k-1})) \\ &\times p(z_k|\mathbf{Y}_{k-1},\mathbf{Z}_{k-1},\mathbf{x}_k)dz_k.\end{aligned}}$$

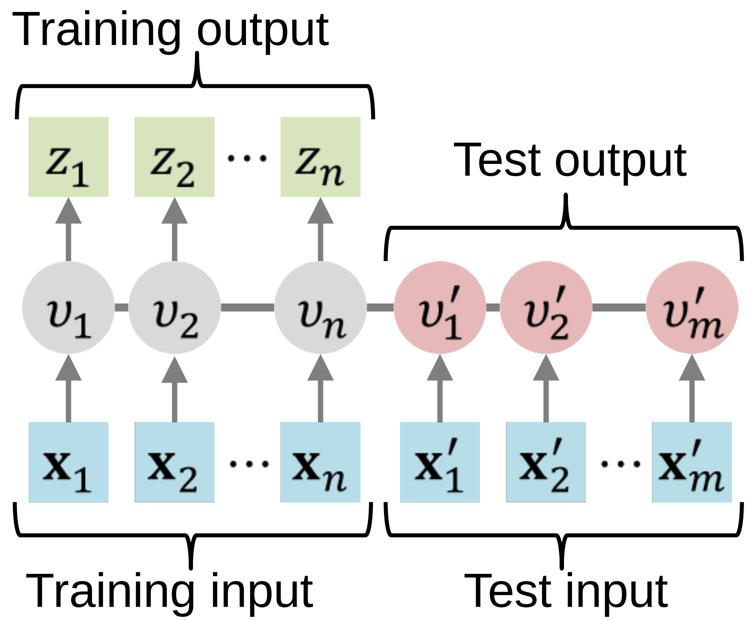
Methodology

Sensor Planning Framework



GP Regression

- GP regression for learning $\mathbf{f}(\cdot)$:
  - Model: $z_i = f(\mathbf{x}_i) + \nu$
  - Training input: $\mathbf{X} = [\mathbf{x}_1^T \quad \cdots \quad \mathbf{x}_n^T]^T$
  - Training output: $\mathbf{z} = [z_1 \quad \cdots \quad z_n]^T$
  - Test input: $\mathbf{X}' = [(\mathbf{x}_1')^T \quad \cdots \quad (\mathbf{x}_m')^T]^T$
  - Test output: $\boldsymbol{\upsilon}' = [\upsilon_1' \quad \cdots \quad \upsilon_m']^T$



- Joint distribution

$$\begin{bmatrix} \mathbf{z} \\ \boldsymbol{\upsilon}' \end{bmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} \boldsymbol{\Phi}(\mathbf{X},\mathbf{X}) + \sigma_n^2 \mathbf{I} & \boldsymbol{\Phi}(\mathbf{X},\mathbf{X}') \\ \boldsymbol{\Phi}(\mathbf{X}',\mathbf{X}) & \boldsymbol{\Phi}(\mathbf{X}',\mathbf{X}') \end{bmatrix}\right)$$

where

$$\boldsymbol{\Phi}(\mathbf{X},\mathbf{X}') \triangleq \begin{bmatrix} \phi(\mathbf{x}_1,\mathbf{x}_1') & \cdots & \phi(\mathbf{x}_1,\mathbf{x}_m') \\ \vdots & \ddots & \vdots \\ \phi(\mathbf{x}_n,\mathbf{x}_1') & \cdots & \phi(\mathbf{x}_n,\mathbf{x}_m') \end{bmatrix}$$

GP Regression Example

- Posterior distribution: $\boldsymbol{\upsilon}' \sim \mathcal{N}(\boldsymbol{\mu}', \boldsymbol{\Sigma}')$, where,

$$\boldsymbol{\mu}' = \boldsymbol{\Phi}\left(\mathbf{X}', \mathbf{X}\right)\left[\boldsymbol{\Phi}(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I}_n\right]^{-1} \mathbf{z}$$

$$\boldsymbol{\Sigma}' = \boldsymbol{\Phi}(\mathbf{X}', \mathbf{X}') - \boldsymbol{\Phi}(\mathbf{X}', \mathbf{X})\left[\boldsymbol{\Phi}(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I}_n\right]^{-1} \boldsymbol{\Phi}(\mathbf{X}, \mathbf{X}')$$

- Example:

$$f(x) = -\cos(x)\sin(y)$$

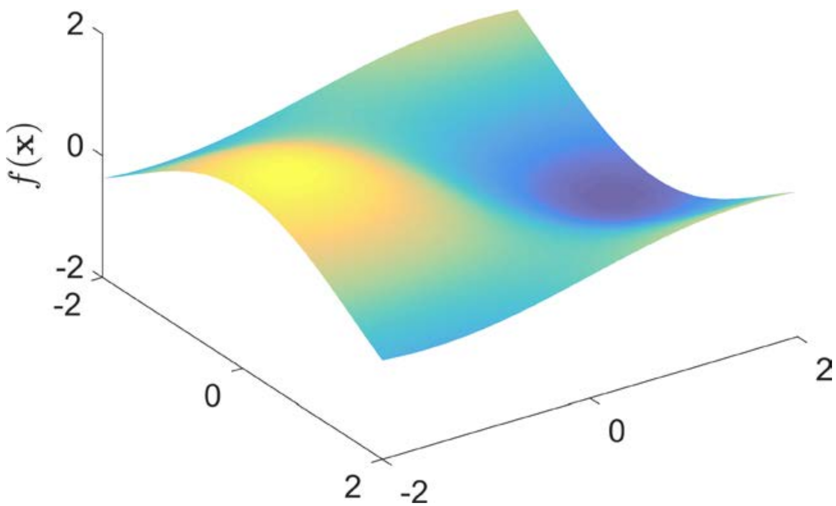


Fig. Ground truth

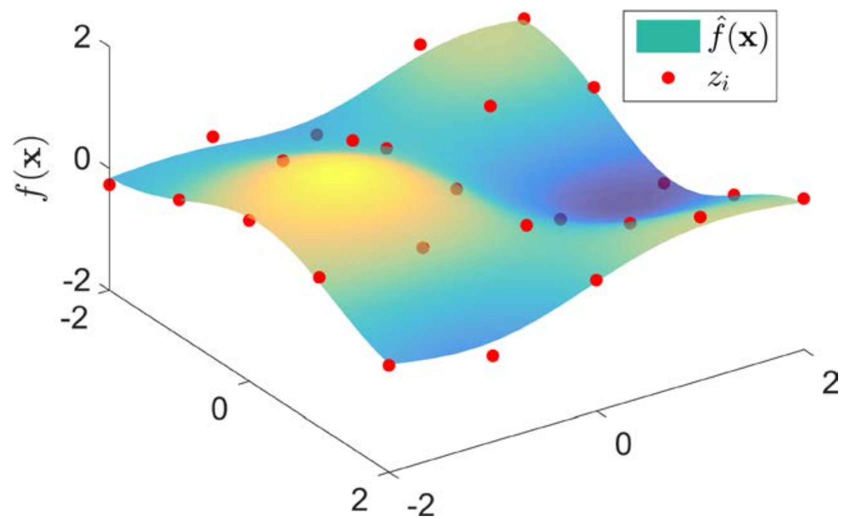
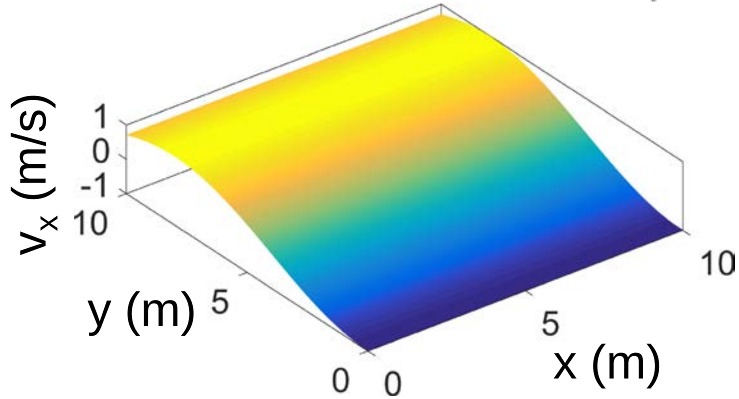


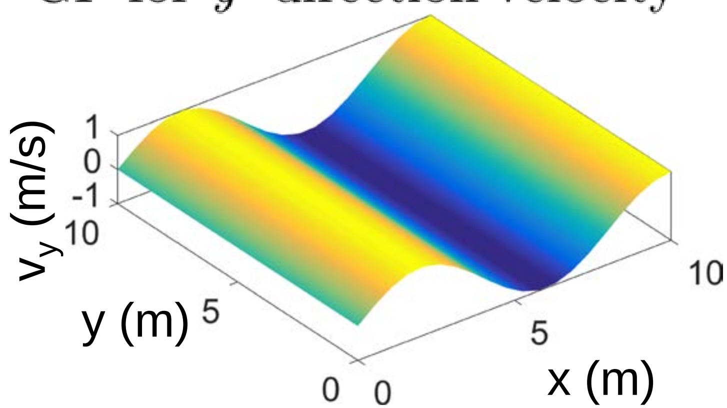
Fig. GP regression result

GP Target Kinematics Model

- Velocity field: $\mathbf{f}(\mathbf{x}) : [x \quad y]^T \rightarrow [v_x \quad v_y]^T$
  - GP for x -direction velocity

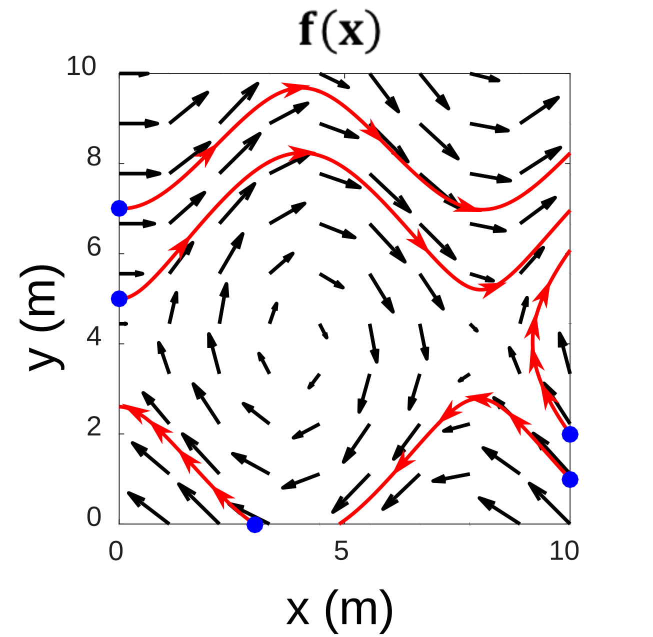


  - GP for y -direction velocity



: Target trajectory

: Target initial position



{ Equivalently: single multi-output GP with diagonal covariance matrix function

GP Information Value

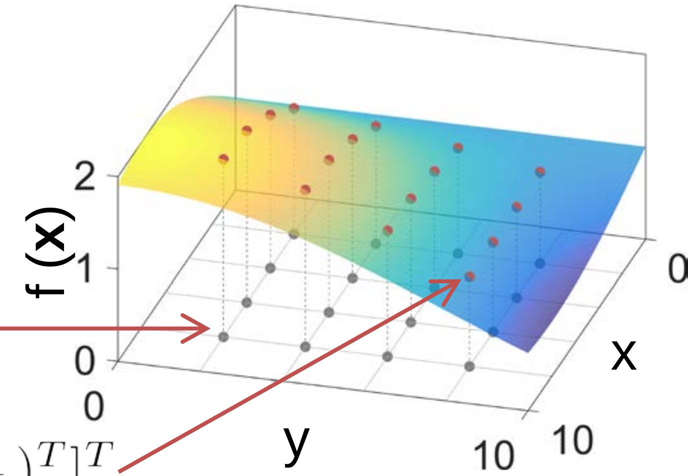
- GP information value: difference between posterior and prior GP

$$D(\boldsymbol{v}_i; \mathbf{m}_j(k+1)) \triangleq D\Big(p(\boldsymbol{v}_i|\mathcal{M}(k+1)) \parallel p(\boldsymbol{v}_i|\mathcal{M}(k))\Big)$$

  - $\mathbf{m}_j(k+1) \triangleq [\mathbf{y}_{j,k+1} \quad \mathbf{z}_{j,k+1}]$: Measurements of position and velocity
  - D: Kullback-Leibler (or other) divergence

$$D\big(p(x) \parallel q(x)\big) = \int_{-\infty}^{\infty} p(x) \ln \frac{p(x)}{q(x)} dx$$

  - Collocation points: $\boldsymbol{\xi} = [\boldsymbol{\xi}_1^T \quad \ldots \quad \boldsymbol{\xi}_L^T]^T$
  - Latent variables: $\boldsymbol{v}_i \triangleq [\mathbf{f}_i(\boldsymbol{\xi}_1)^T \quad \cdots \quad \mathbf{f}_i(\boldsymbol{\xi}_L)^T]^T$



  - Measurement history: $\mathcal{M}(k) \triangleq \cup_{j=1}^{N}\{\mathbf{m}_j(\ell) \mid 1 \leq \ell \leq 1\}$

- Problem: future measurement, $\mathbf{m}_j(k+1)$, is unknown

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GP-EKLD

- GP expected KL divergence (GP-EKLD)

$$\hat{D}(\boldsymbol{v}_i; \mathbf{m}_j(k+1)) = \int_{\mathbb{R}} D(\boldsymbol{v}_i; \mathbf{m}_j(k+1)) p(\mathbf{m}_j(k+1)) d\mathbf{m}_j(k+1)$$

- **(Theorem GP-EKLD)** The GP-EKLD affords the analytical solution,

$$\hat{D} = \frac{1}{2}\left[\mathrm{tr}\left(\boldsymbol{\Sigma}_{i,k}^{-1}\boldsymbol{\Sigma}_{i,k+1} + \mathbf{Q}^{-1}\mathbf{R}^T\boldsymbol{\Sigma}_{i,k}^{-1}\mathbf{R}\mathbf{Q}^{-1}\sigma_v^2\right) - \ln\left(\frac{|\boldsymbol{\Sigma}_{i,k+1}|}{|\boldsymbol{\Sigma}_{i,k}|}\right) - 2L\right]$$

where

$\mathbf{Y}_{i,k}$: All target position measurements of ith VF

$$\boldsymbol{\Sigma} \triangleq \boldsymbol{\Phi}(\mathbf{Y}_{i,k}, \mathbf{Y}_{i,k}) + \sigma_v^2\mathbf{I}$$

$$\boldsymbol{\Sigma}_{i,k} \triangleq \boldsymbol{\Phi}(\boldsymbol{\xi}, \boldsymbol{\xi}) - \boldsymbol{\Phi}(\boldsymbol{\xi}, \mathbf{Y}_{i,k})\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}(\mathbf{Y}_{i,k}, \boldsymbol{\xi})$$

$$\mathbf{R} \triangleq \boldsymbol{\Phi}(\boldsymbol{\xi}, \mathbf{y}_{j,k+1}) - \boldsymbol{\Phi}(\boldsymbol{\xi}, \mathbf{Y}_{i,k})\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}(\mathbf{Y}_{i,k}, \mathbf{y}_{j,k+1})$$

$$\mathbf{Q} \triangleq \boldsymbol{\Phi}(\mathbf{y}_{j,k+1}, \mathbf{y}_{j,k+1}) - \boldsymbol{\Phi}(\mathbf{y}_{j,k+1}, \mathbf{Y}_{i,k})\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}(\mathbf{Y}_{i,k}, \mathbf{y}_{j,k+1}) + \sigma_v^2\mathbf{I}$$

LABORATORY FOR INTELLIGENT SYSTEMS AND CONTROLS

DPGP-EKLD

- DPGP expected KL divergence (DPGP-EKLD):

$$\hat{D}\left(\boldsymbol{v};\mathbf{m}_j(k+1)\right)=\mathbb{E}_{\mathbf{m}_j(k+1)}\left\{\mathbb{E}_{G_j(k+1)}\left\{D\left(\boldsymbol{v};\mathbf{m}_j(k+1)\right)\right\}\right\}$$

  - Difference between posterior and prior DPGP
  - Collocation points: $\boldsymbol{v}\triangleq[\boldsymbol{v}_1^T \quad \cdots \quad \boldsymbol{v}_M^T]^T$
  - G_j: Target-VF association does not change
  - Measurement consistent with GP regression

- **(Theorem DPGP-EKLD)** The DPGP-EKLD can be simplified as,

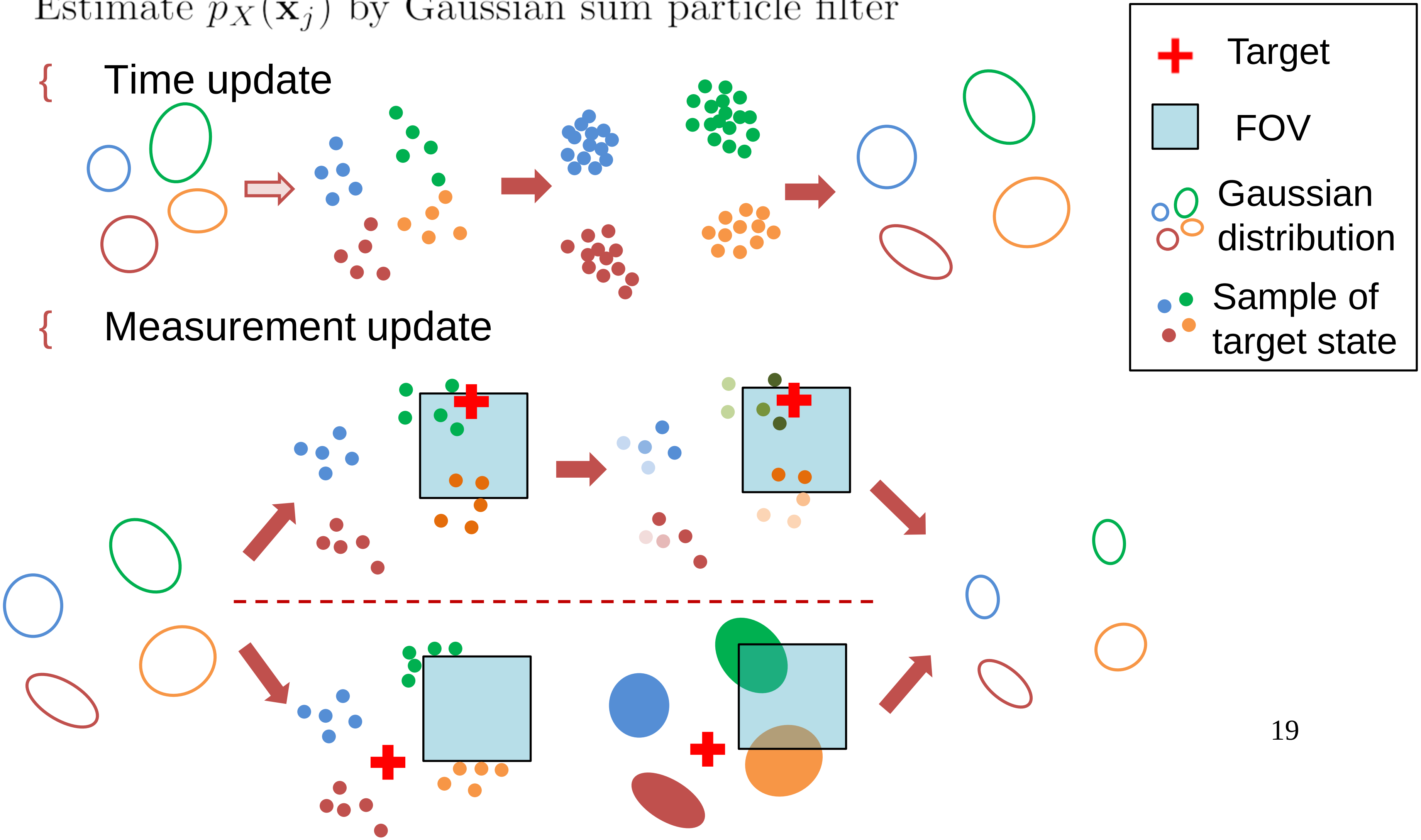
$$\hat{D}\big(\boldsymbol{v};\mathbf{m}_j(k+1)\big)=\sum_{i=1}^{M}w_{ij}\int_{\mathcal{S}}h_i[\mathbf{x}_j(k+1)]\times p_X\left(\mathbf{x}_j(k+1)\right)d\mathbf{x}_j$$

where $h_i[\mathbf{x}_j(k+1)]=\hat{D}\big(\boldsymbol{v}_i;\mathbf{m}_j(k+1)\big)$

- Problem: $p_X\left(\mathbf{x}_j(k+1)\right)=\int p_V\left(\frac{\mathbf{x}_j(k+1)-\mathbf{x}_j(k)}{\Delta t}\right)p_X\big(\mathbf{x}_j(k)\big)d\mathbf{x}_j$

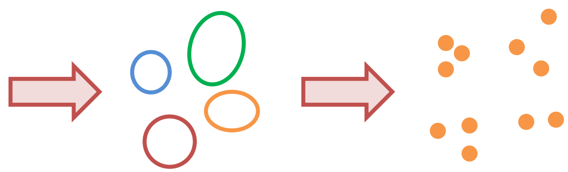
DPGP Particle Filter

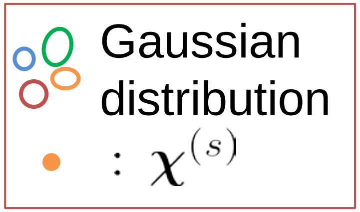
- Special case: $p_X\left(\mathbf{x}_j(k+1)|\mathbf{x}_j(k)\right)$ is Gaussian distributed
- Estimate $p_X(\mathbf{x}_j)$ by Gaussian sum particle filter
  - Time update
  - Measurement update



DPGP-EKLD Approximation

- Approximation of DPGP-EKLD:

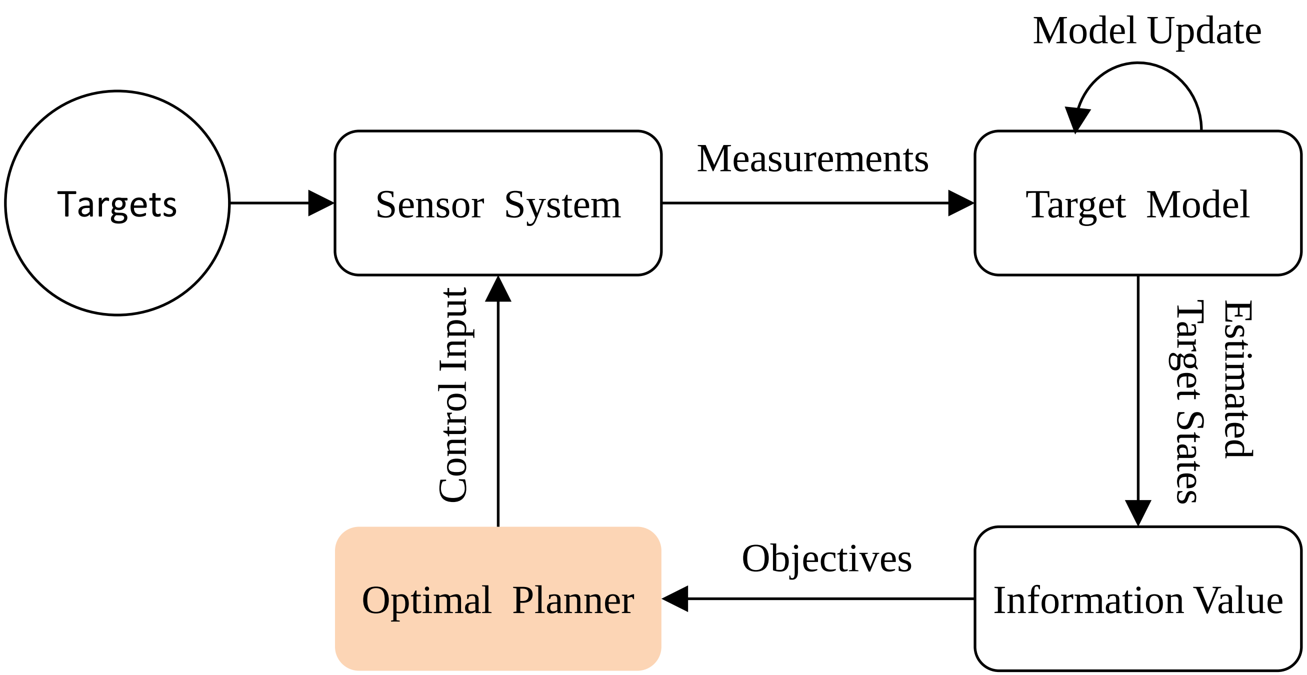
$p_X\left(\mathbf{x}_j(k+1)\right)$ 



$$\hat{D}\left(\boldsymbol{v};\mathbf{m}_j(k+1)\right) \approx \sum_{i=1}^{M} \frac{w_{ij}}{S} \sum_{\chi^{(s)}\in\mathcal{S}} h_i(\chi^{(s)})$$

- (Lemma 1) $\mathbf{Q}$ is bounded: $\phi_0 + \frac{\sigma_v^2}{k\ \mathrm{tr}(\phi_0)+\sigma_v^2}\mathbf{I}_2 \preceq \mathbf{Q} \preceq \phi_0 + \sigma_v^2\mathbf{I}_2$
- (Lemma 2) $\mathbf{\Sigma}_{i,k+1}$ is bounded: $\mathbf{0} \prec \mathbf{\Sigma}_{i,k+1} \preceq \mathbf{\Sigma}_{i,k}$
- (Lemma 3) $\mathbf{R}$ is bounded: $0 \leq \mathrm{tr}(\mathbf{R}\mathbf{R}^T) \leq 4k[(\phi_0) + 2\sigma_v^2]$
- (Lemma 4) $h_i(\cdot)$ is bounded
- **(Theorem DPGP-EKLD Approximation)**
  - The Monte Carlo integration is unbiased
  - The variance decreases linearly with the number of samples

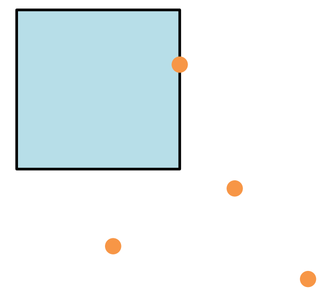
Methodology Part III: Sensor Planning Algorithms

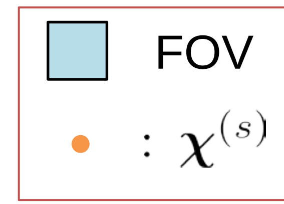


Optimize DPGP-EKLD

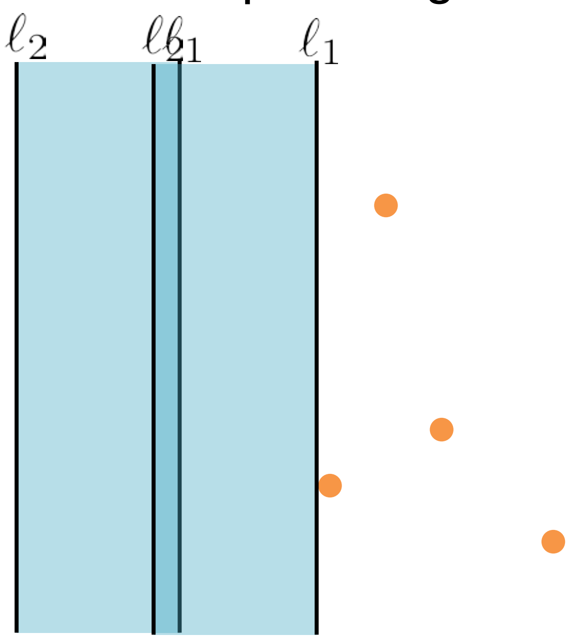
- DPGP-EKLD optimization without sensor dynamics constraints
- DPGP-EKLD approximation:

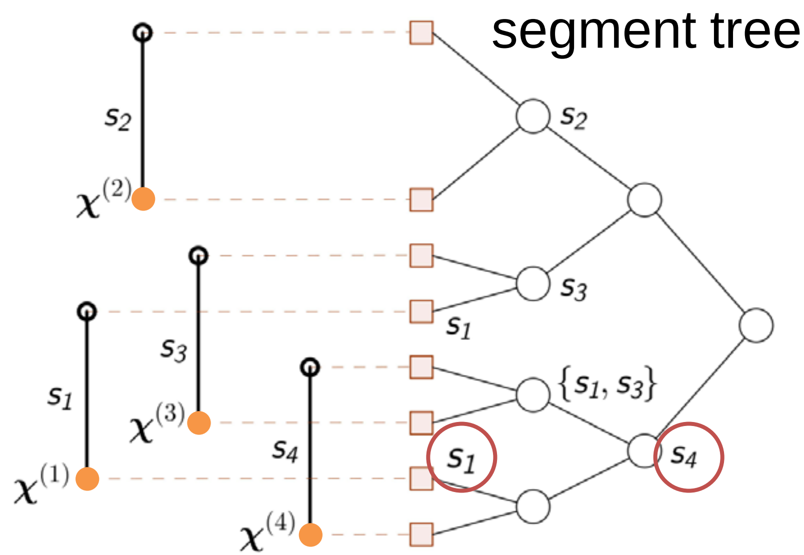
$$\sum_{\chi^{(s)}\in\mathcal{S}(k)} \sum_{i=1}^{M} \frac{w_{ij}}{S} h_i\left(\chi^{(s)}\right)$$





- Sweep line algorithm:





Incorporate Sensor Dynamics

- DPGP-EKLD optimization with sensor dynamics constraints
- Linear sensor dynamics with constraints:

$$\begin{aligned} \mathbf{s}(k+1) &= \mathbf{A}\mathbf{s}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{s}(k) \in \mathcal{A} \quad &\text{and} \quad \mathbf{u}(k) \in \mathcal{U} \end{aligned} \tag{1}$$

- Lower bound: $(1-\gamma)\sum_{\ell=k}^{k'} \gamma^{\ell-k}\hat{D}(\boldsymbol{v}_i;\mathbf{m}_j(\ell))$
- Reward for observing jth target that follows ith VF:

$$J_{ij} \triangleq w_{ij}\sum_{\ell=k}^{k'}(1-\gamma)\gamma^{\ell-k}\hat{D}(\boldsymbol{v}_i;\mathbf{m}_j(\ell))P(\mathbf{s}(\ell),\mathbf{x}_j(\ell))$$

- Multiple objective optimization

$$\begin{aligned} &\underset{\mathbf{u}(\ell),\ k\le\ell\le k'}{\text{maximize}} \quad \begin{bmatrix} J_{11} & \cdots & J_{MN} \end{bmatrix}^T \\ &\text{subject to} \quad (1) \end{aligned}$$

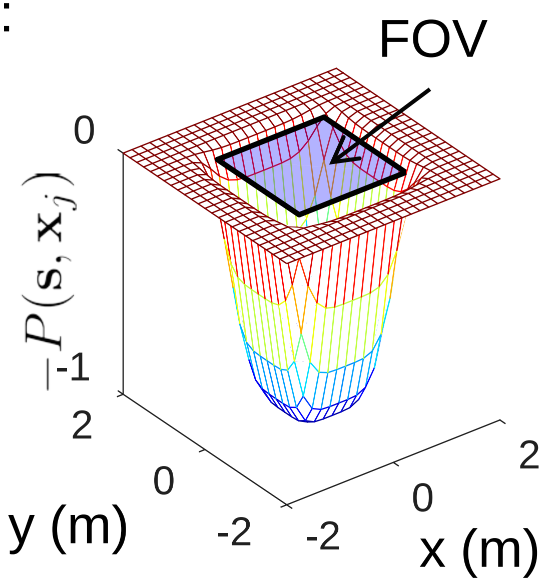


Fig. Example of $P(\mathbf{s},\mathbf{x}_j)$

Lexicographic Algorithm

- Objectives can be ordered by relative importance

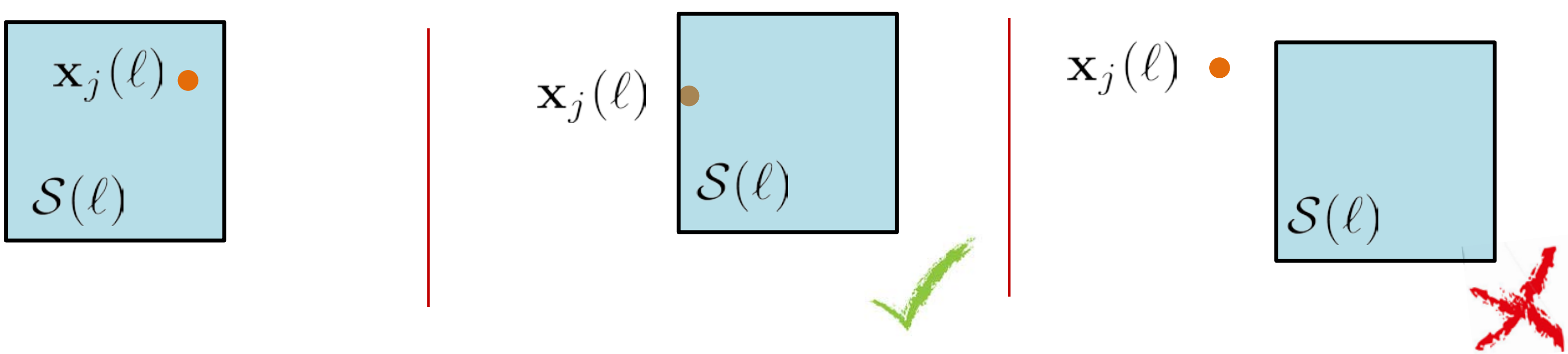
$$\cup_{i=1}^{M} \cup_{j=1}^{N} \{J_{ij}\} \xrightarrow{\text{reorder}} \cup_{\imath=1}^{MN} \{J'_\imath\}$$

- Optimize ordered objectives sequentially: For $\imath = 1, \ldots, MN$,

$$\begin{aligned} &\underset{\mathbf{u}(\ell),\ k \leq \ell \leq k'}{\text{maximize}} && J'_\imath \\ &\text{subject to} && J'_\jmath \geq (J'_\jmath)^*,\ \jmath = 1, \ldots, \imath - 1 \end{aligned}$$

- First iteration: convex optimization with linear constraints

- Remaining iterations: Additional constraints:



- Geometric constraint on sensor state: $\mathbf{s} \in \mathcal{T}(\mathbf{x}_j)$

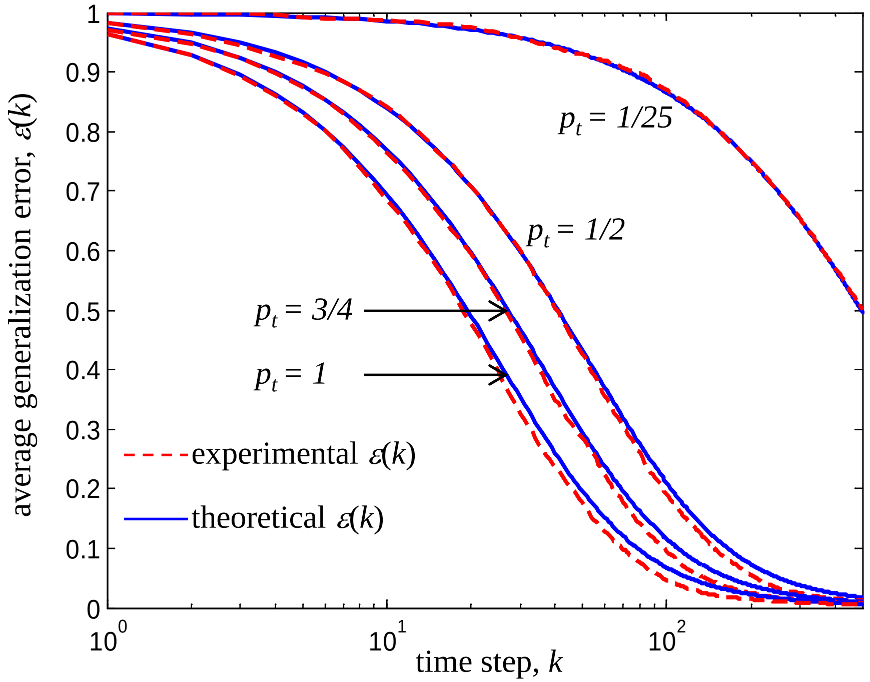
Nominal Network Performance

- Let Σ denote the covariance matrix and $\Psi(\mathbf{x}, \mathbf{y})$ denote the cross-covariance matrix, then the GP **average generalization error** (AGE),

$$\varepsilon(k) = \mathrm{E}_{\mathbf{x}}\left\{ \Sigma(\mathbf{x}) - \Psi(\mathbf{x}, \mathbf{Y}_k)\left[\Sigma(\mathbf{Y}_k) + \delta^2 \mathbf{I}\right]^{-1} \Psi^T(\mathbf{x}, \mathbf{Y}_k) \right\}$$

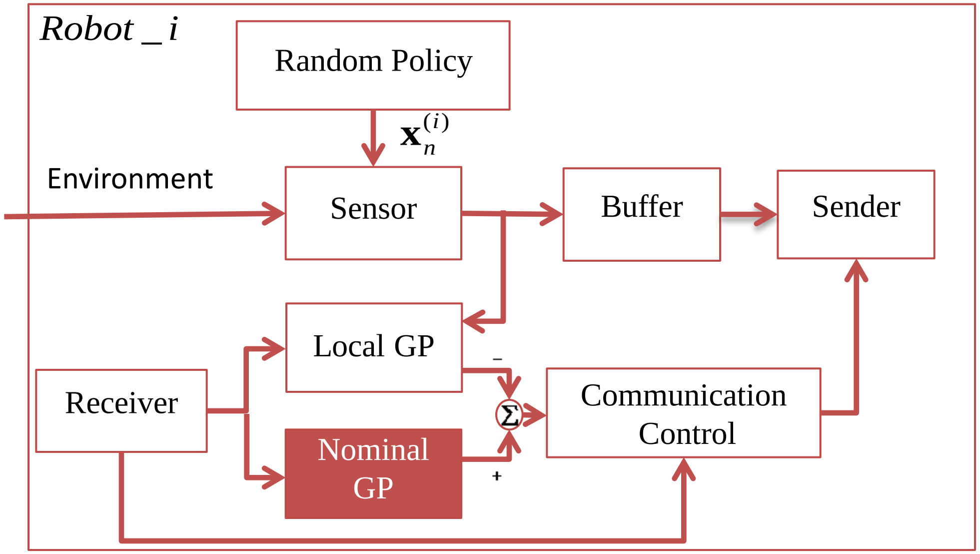
represent a measure of GP performance.

- From the latest GP, the posterior covariance, and the network nominal AGE can be estimated from an assumed probability distribution for future measurement locations, and an assumed probability of detection p_t



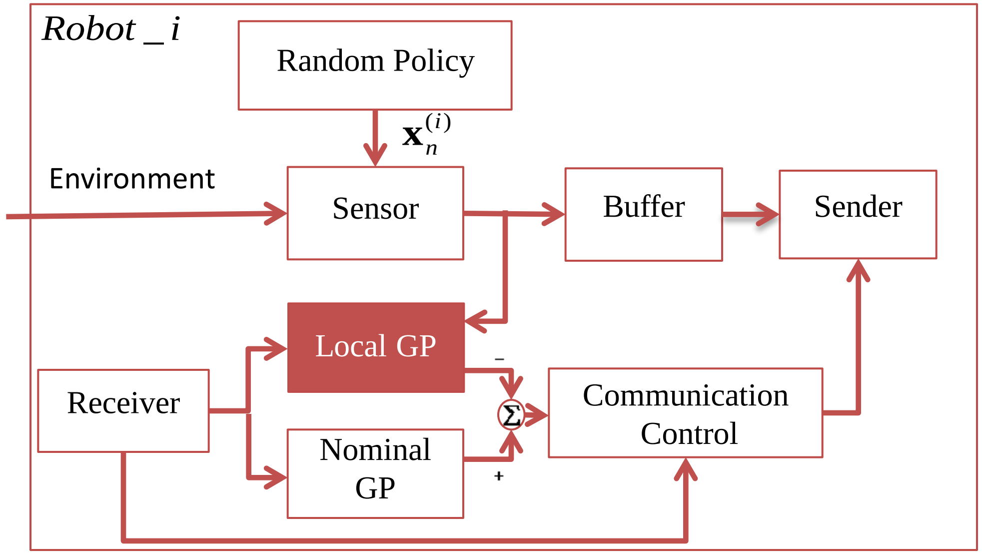
AGE Communications

Communication Control



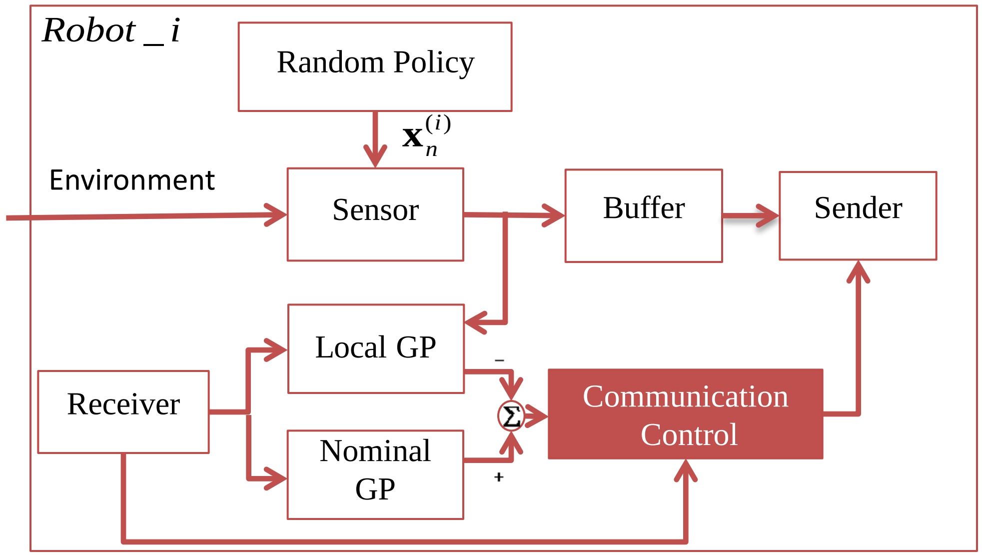
- Approximate nominal average generalization error (AGE) from latest GP

Communication Control



- Approximate nominal average generalization error (AGE) from latest GP
- Local GP (GPL) computation
  - GPL is updated using local measurements (obtained by robot *i*)
  - Actual AGE is calculated from the local covariance function.

Communication Control

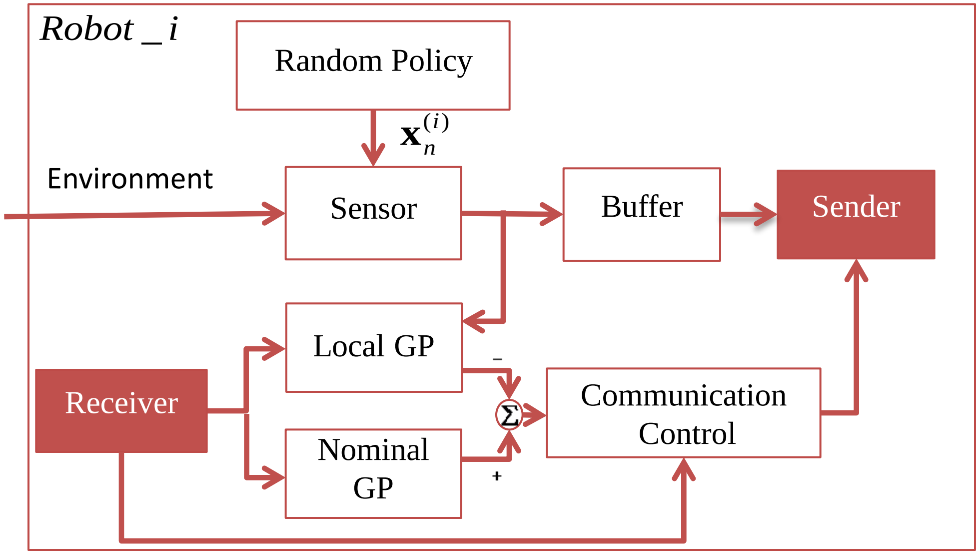


- Approximate nominal average generalization error (AGE) from latest GP
- Local GP (GPL) computation
- Communication time: at the n^{th} time step, the i^{th} robot communicates if

$$\max_i \left| \sum_{k=n_0}^{n} \varepsilon_i(k) - \varepsilon_{\text{nominal}}(k) \right| > \gamma$$

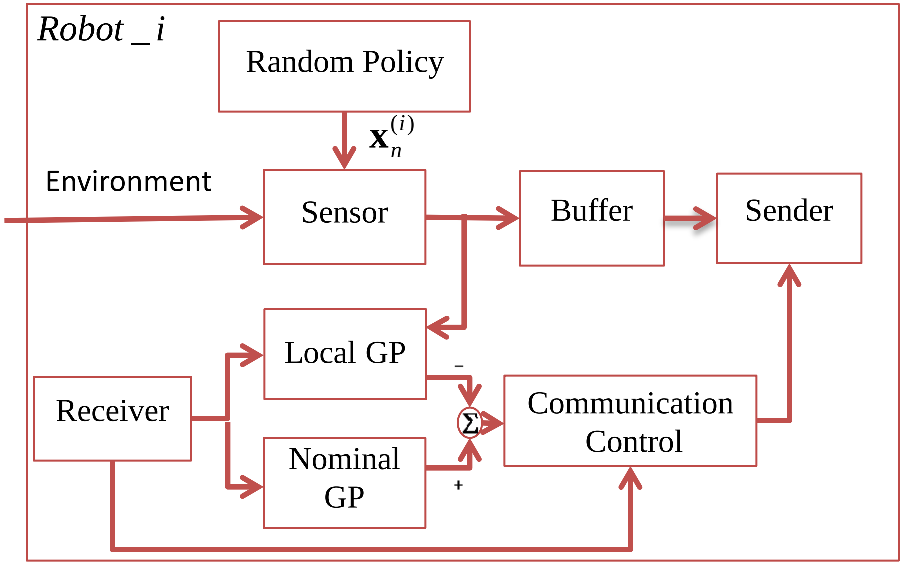
where, γ is predefined performance threshold.

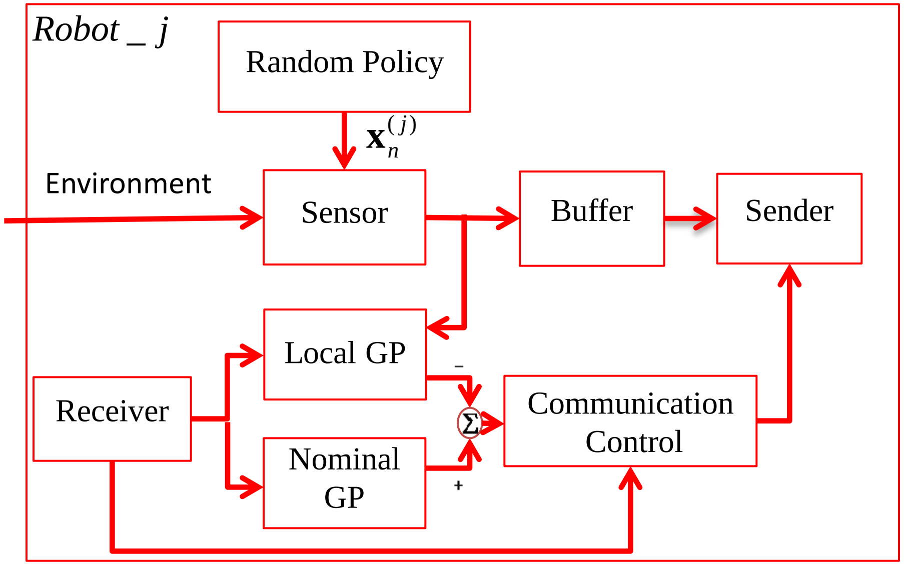
Communication Control

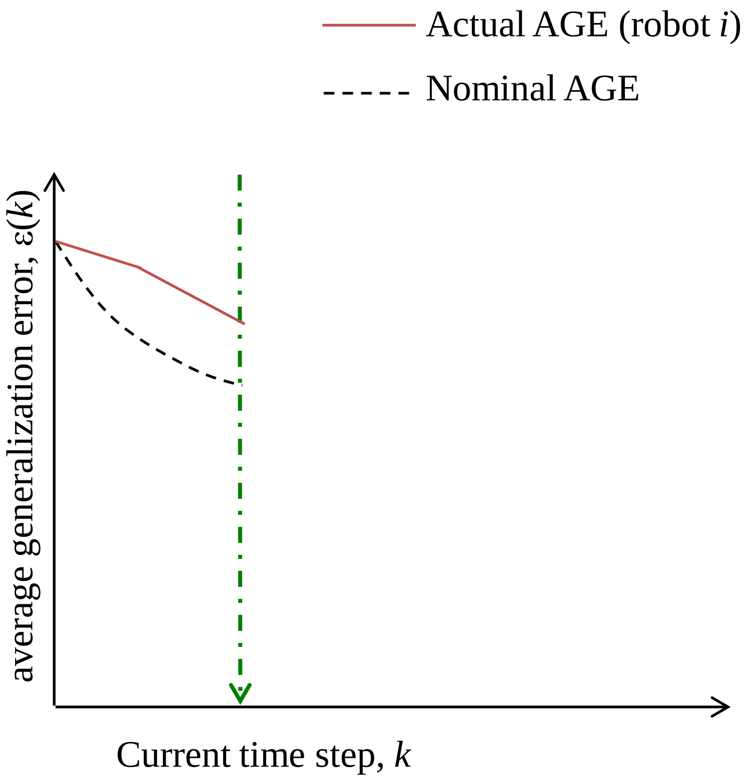


- Approximate nominal average generalization error (AGE) from latest GP
- Local GP (GPL) computation
- Communication time
- Information sharing: all new measurements are communicated and used to update the robot GP

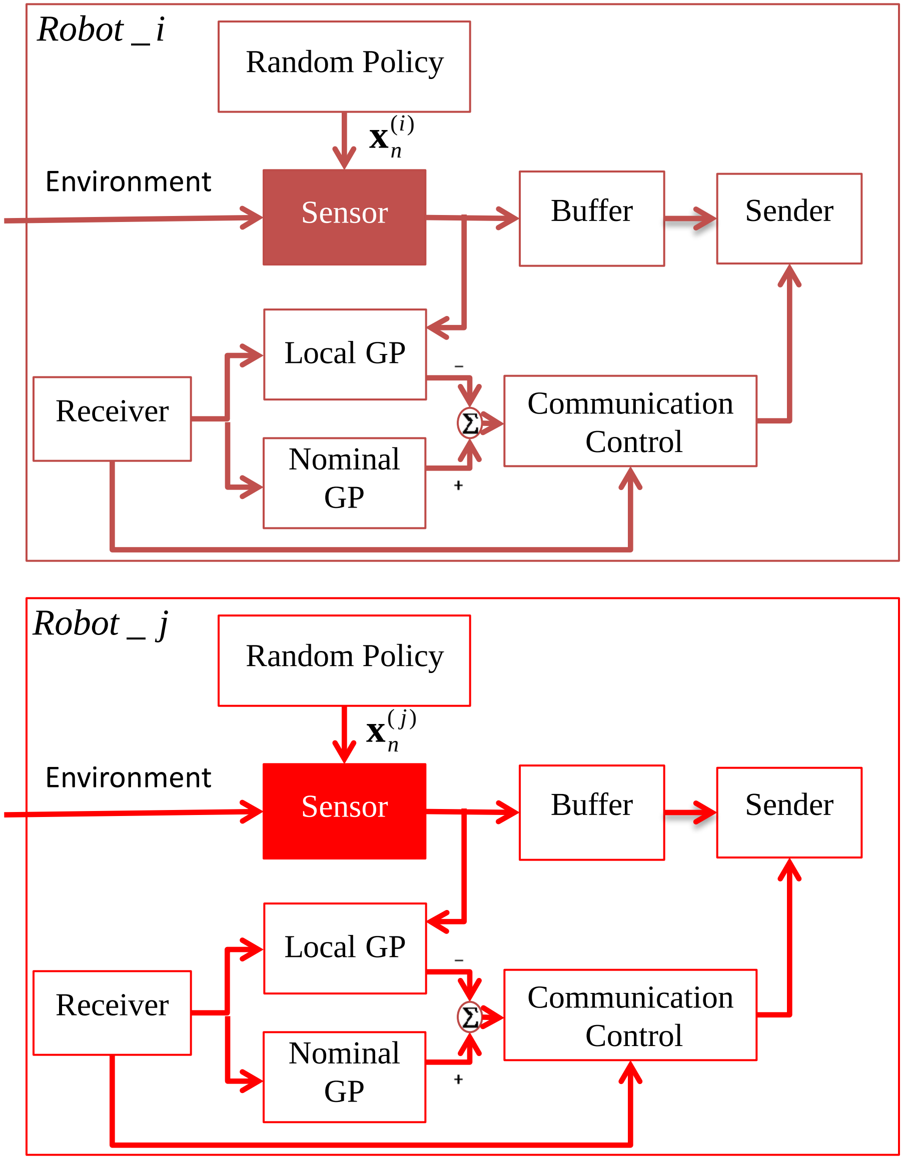
Communication Control

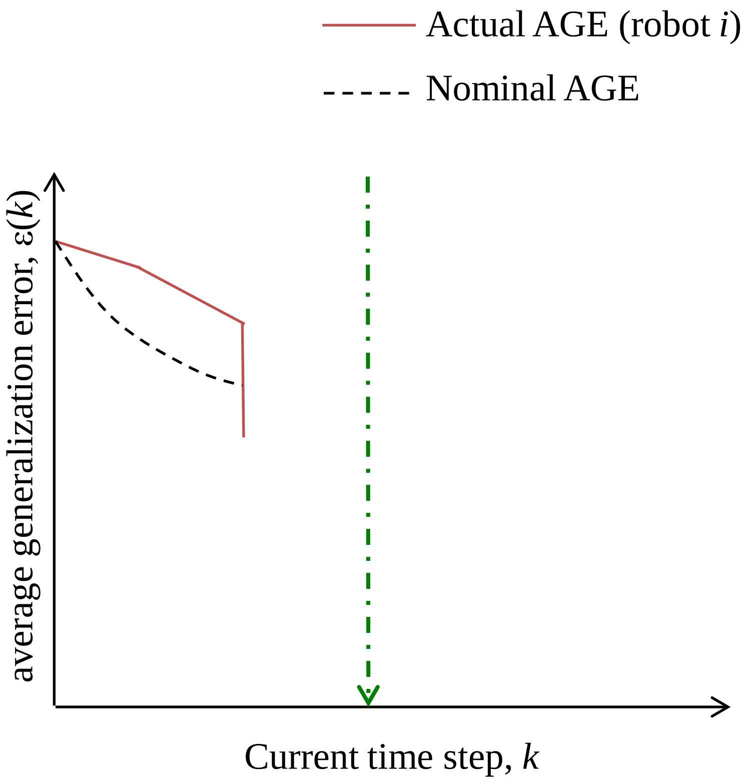




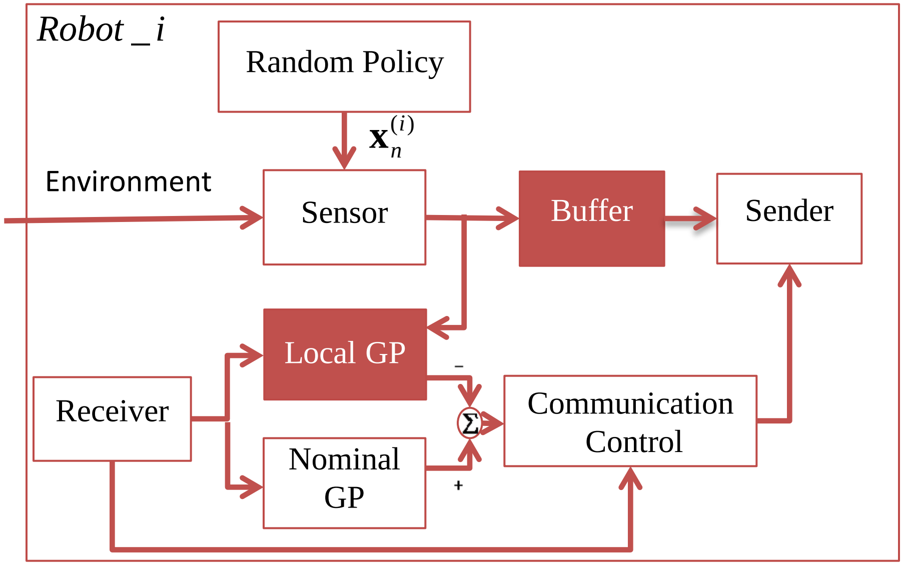


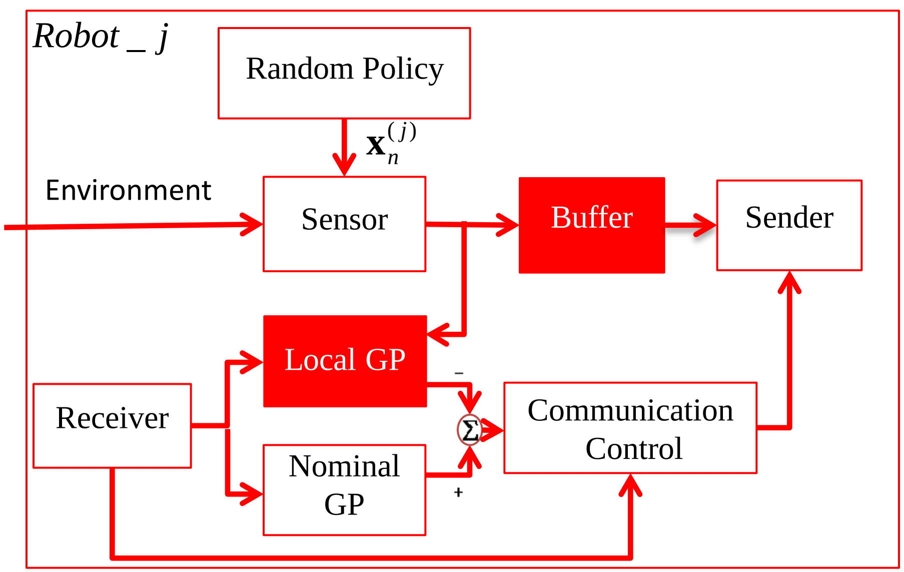
Communication Control

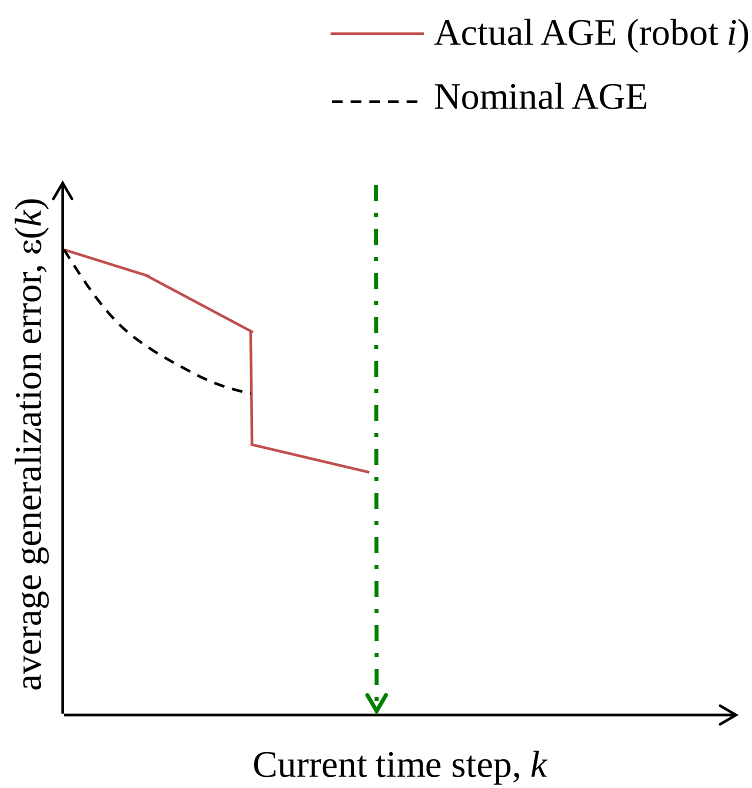




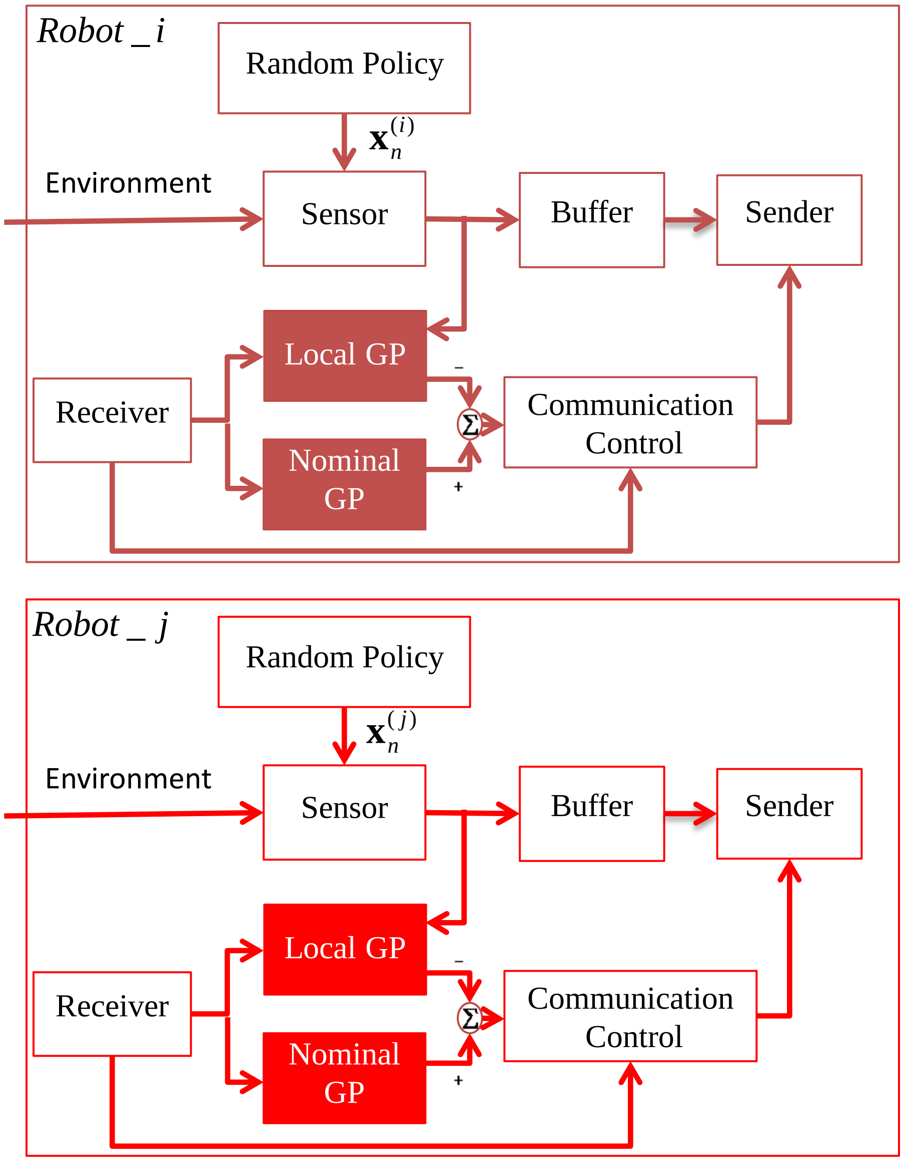
Communication Control

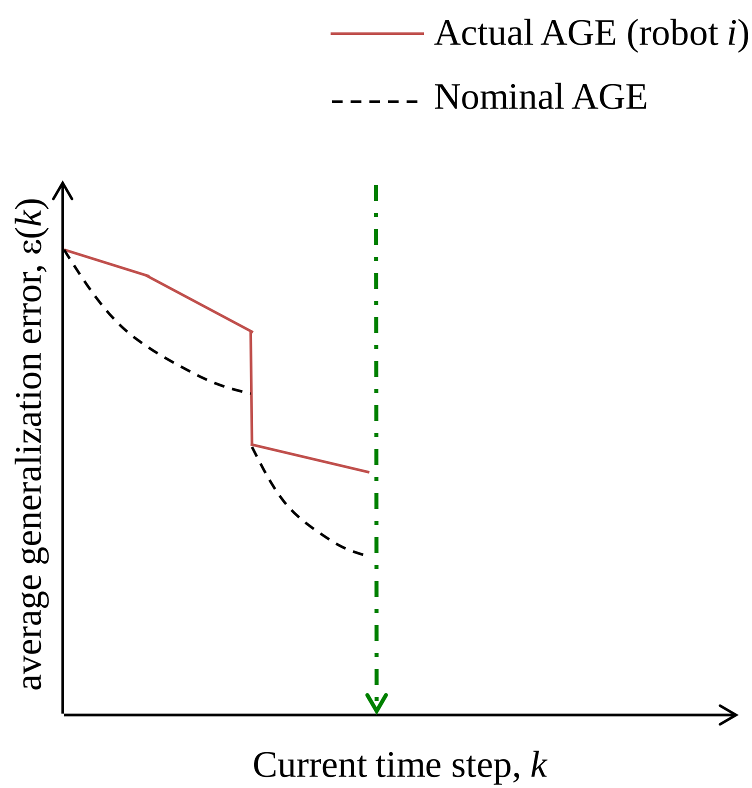






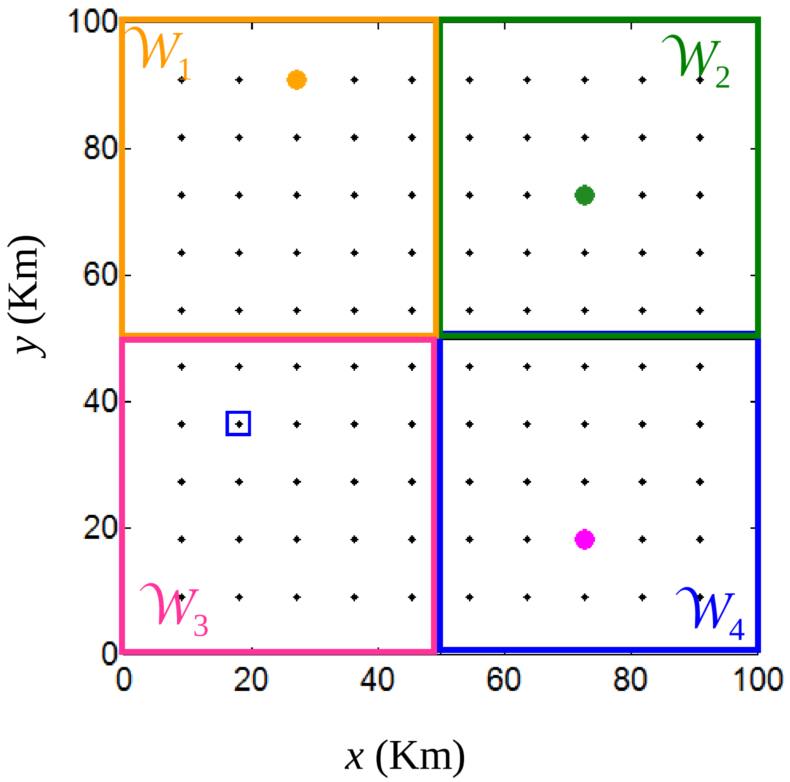
Communication Control

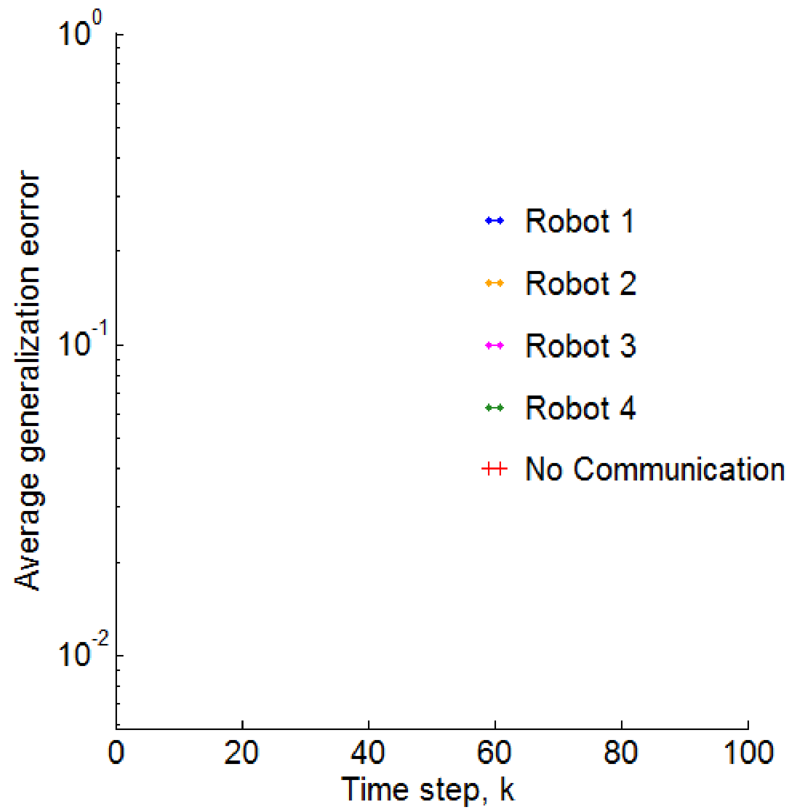




Simulation Results

Modeling of a spatial phenomenon, $g(\mathbf{x})$, by four robots with disjoint workspaces:





LABORATORY FOR INTELLIGENT SYSTEMS AND CONTROLS

Conclusions

- Conclusion
  - Developed novel information theoretic functions for GP models of distributed spatio-temporal processes
  - Developed information-theoretic sensor planning algorithms for distributed networks with bounded fields of view
  - Developed AGE method for monitoring information sufficiency and schedule communications
- Future work
  - Extend information theoretic functions to partially observable targets
  - Extend information theoretic functions to decentralized control
  - Develop multiscale adaptive planners to uncertain environments

Thank you!