Biophysical Modeling of Satisficing Control Strategies as Derived from Quantification of Primate Brain Activity and Psychophysics

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Introduction and Motivation

Satisficing in Humans and Primates

- *Bounded* rationality
- Accessing stored solutions
- Satisficing strategies for information gathering
- *Ecological* rationality – adaptive decision-making and heuristics

Satisficing Decision Strategies

- Noisy and incomplete information
- Insufficient intelligence
- Time pressure
- Utility cannot be maximized
- Consequences and probabilities may not be learned
Technical Accomplishments – Year 1

- Demonstrated systematic framework for biophysical modeling of satisficing strategies for inferential decision making
- Developed graphical model (BN) formulation of passive satisficing search problems
- Demonstrated integrated approach to multidisciplinary human studies and experiments via online crowdsourcing (www.mturk.com)
- Completed human and non-human primate (NHP) behavioral studies on passive satisficing task
- Developed satisficing strategies for passive search problems with time pressure / limited data / incomplete probabilistic model
- Developed human-inspired anytime BN learning algorithm
- Tested BN learning algorithm with empirical and virtual datasets
Technical Accomplishments – Year 2

- Developed a graphical model formulation and Webot software environment for *active and dynamic* satisficing search problems
- Started to establish the biophysical basis of satisficing using fMRI in humans and single neuron recordings in NHPs
- Developed adaptive evidence accumulation (AEA) algorithm for satisficing decision making under time pressure and with limited data
- Tested and demonstrated AEA effectiveness on empirical and virtual benchmark datasets

**Outcomes and Deliverables:**

- Two journal manuscripts in preparation(review
- Three conference abstracts
- Immersive environment Webot-based software
Approach
Approach: Biophysical Modeling

Experiments:

I. Behavioral Studies

II. Human (fMRI) Neuro-imaging and –stimulation Studies

III. Non-human Primates Neural Recording Studies
Satisficing Tasks and Experiments

1. Passive Satisficing

Learn and apply predictive information from given cues

2. Active Satisficing

Active information gathering and decision making

- Limited information
- Time pressure
- Cue redundancy
3. Immersive Satisficing (DiVE – ML2VR)

Navigate and discover multiple treasures in complex environments

- Non-compensatory cues
- Risk and environmental pressures
- Semantically meaningful cues (context)
- Interrupted sensory signals
- Bounded controls
Immersive Satisficing Tasks and Experiments
Immersive Environment

- Active and dynamic satisficing tasks: treasure hunt
- Ordering constraints on cue discovery (satisficing tree)
- Complex environment (clutter, obstacles, limited visibility…)

3D Indoor Navigation:

Supervisor Node for Exploration:
Immersive Satisficing Environment via Webot-software
Immersive Environment Robotic Capabilities

- MATLAB® Interface
- Robots and sensors simulations
- 3D navigation and perception
Immersive Environment

• Immersive environment is created in Webots software

• Obstacles are located at the fixed locations in the environment including table, sofa, chairs, lights and so on
Problem Formulation

Problem assumption:

- Assume the immersive environment as workspace \( W \subset \mathbb{R}^2 \)
- Obstacles \( \mathcal{B} = \{ B_1, \ldots, B_m \} \) are placed at fixed locations
- Targets \( \mathcal{T} = \{ T_1, \ldots, T_n \} \) are placed at random locations at each trial
- A robotic sensor with platform geometry \( A \), camera FOV geometry \( s_2 \subset \mathbb{R}^2 \), and measurement FOV geometry \( s_1 \subset s_2 \subset \mathbb{R}^2 \)
- Assume the map is known to the robotic sensor a priori, including estimated geometries and locations of the obstacles

Objectives:

- Avoid all obstacles
- Classify all targets while:
  - Minimizing distance of robot sensor travelling
  - Maximize reward by making as many as correct classification
Robot Dynamics and Configuration Space

Robot Dynamics

- Let $\mathbf{q} = [x \ y \ \theta] \in \mathcal{W}$ be a configuration vector with the position and orientation.
- Let the configuration space $\mathcal{C}$ denote the space of all possible robot configurations.
- Let $\mathcal{CB}$ denote the set of configurations that cause collisions between the platform $\mathcal{A}$ and obstacles in $\mathcal{B}$.
- Let $\mathcal{C}_{\text{free}} = \mathcal{C} \setminus \mathcal{CB}$ denote the set of configurations where no such collisions occur.

The configuration vector $\mathbf{q}$ must satisfy the dynamic equation of the robotic sensor given by differential drive kinematics:

$$
\mathbf{q} = \begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
\frac{r}{2}(u_i + u_r)\cos \theta \\
\frac{r}{2}(u_i + u_r)\sin \theta \\
\frac{r}{L}(u_r - u_i)
\end{bmatrix} = f(\mathbf{q}, \mathbf{u})
$$

where $r$ is the wheel radius and $L$ is the distance between wheels.

The control vector $\mathbf{u} = [u_i \ u_r]^T \in \mathcal{U}$ and control input space $\mathcal{U}$.
Target Classification

Classification model assumption:

- Decision Rule:

  Treasure: \( P(Z_i = 'Treasure' \mid X_{i1}...X_{ik}) > P(Z_i = 'NotTreasure' \mid X_{i1}...X_{ik}) \)

  Not Treasure: \( P(Z_i = 'Treasure' \mid X_{i1}...X_{ik}) < P(Z_i = 'NotTreasure' \mid X_{i1}...X_{ik}) \)

- The measurements are obtained in a fixed order (sequential measurements)

\[
P(Z_i = 'Treasure' \mid X_{i1}...X_{ik}) = \frac{P(X_{i1})P(X_{i2} \mid X_{i1})...P(X_{ik} \mid X_{i1}...X_{i(k-1)})P(Z_i = 'Treasure' \mid X_{i1}...X_{ik})}{P(X_{i1}...X_{ik})}
\]

- The posterior probabilities of class are updated by more measurements

- Assume that all conditional probabilities are known
• In each trial, targets $T = \{T_1, \ldots, T_n\}$ are generated from the BN model.

• Each target $T_i$ has $K=3$ layers of features/cues: $\{x_{ik}\}_{k=1,2,3}$ and classification $Z_i$.

• Generated targets are located in the Webots environment for experiments.

• Locations of targets in the workspace are generated from a uniform distribution on a set of possible locations.
Statistics Model

Conditional probability parameters of BN model

<table>
<thead>
<tr>
<th>Probabilities</th>
<th>$P(X_{i1})$</th>
<th>$0.49$</th>
<th>$0.51$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X_{i2}</td>
<td>X_{i1})$</td>
<td>$0.60$</td>
<td>$0.40$</td>
</tr>
<tr>
<td>$P(X_{i3}</td>
<td>X_{i2}X_{i1})$</td>
<td>$0.98$</td>
<td>$0.02$</td>
</tr>
</tbody>
</table>

Targets and corresponding classes

<table>
<thead>
<tr>
<th>Target</th>
<th>Class  $z_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple</td>
<td>Not Treasure</td>
</tr>
<tr>
<td>Watermelon</td>
<td>Treasure</td>
</tr>
<tr>
<td>Orange</td>
<td>Not Treasure</td>
</tr>
<tr>
<td>Basketball</td>
<td>Treasure</td>
</tr>
<tr>
<td>Cardboard Box</td>
<td>Not Treasure</td>
</tr>
<tr>
<td>Wooden Box</td>
<td>Not Treasure</td>
</tr>
<tr>
<td>Computer</td>
<td>Not Treasure</td>
</tr>
<tr>
<td>Book</td>
<td>Treasure</td>
</tr>
</tbody>
</table>
Target locations

- Possible locations of targets (blue stars) and fixed locations of obstacles (black squares) in the left figure.
- Locations of targets (green circles in the right figure) in the workspace are generated from a uniform distribution on a set of possible locations.
- The robot (red circle and FOV) is initially located at the middle point in the environment.
Webots GUI

Learning Phase

- No cost
- No time limitation
- No limited money balance

Testing Phase

- Limited cost
- Time limitation
- Limited money balance
Robot Movements

The robotic sensor navigates the workspace $w$ through pure translation and pure rotation. Therefore, the robot movements can be expressed by the commands “Move Forward, Move Back, Rotate Left, and Rotate Right”.

The robot state equation can be rewritten as follows:

\[
\dot{\mathbf{q}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} ru_r \cos \theta \\ ru_r \sin \theta \\ \frac{r}{L} u_\Omega \end{bmatrix} = f(\mathbf{q}, \mathbf{u})
\]

translational velocity (a): $u_T = (u_r + u_l)/2$

rotational velocity (b): $u_\Omega = u_r - u_l$

control velocity: $\mathbf{u} = [u_T, u_\Omega]^T$

Both translational and rotational velocities are ranged such as

\[
u_T \in [-u_{\text{max}}, u_{\text{max}}] \quad u_\Omega \in [-\omega_{\text{max}}, \omega_{\text{max}}]
\]
Probabilistic Roadmap Method (PRM)

- A set of nodes or milestones $V = [v_1, ..., v_N]$ are sampled randomly in $C_{free}$
- Edges $E = [(v_1, v_2), ..., (v_{N-1}, v_N)]$ are simple local paths connecting $v_i$ and $v_j$
- Nodes $V$ and edges $E$ form an undirected graph $G = (V, E)$ as roadmap

Nodes/milestones random sampling

Nodes are sampled using a weighted PDF $\pi$

$$\pi = w_2 \pi_V + w_1 (1 - w_2) \pi_G + (1 - w_1)(1 - w_2) \pi_U \quad 0 \leq w_1, w_2 \leq 1$$

- A uniform PDF $\pi_U$ in the workspace
- A PDF $\pi_G$ for sampling narrow regions using the bridge test
- A PDF $\pi_V$ for sampling regions of high information value (not applied in this project)
Path Planning

Greedy path planning

- Apply a heuristic weighted cost function

\[ C(v_i, v_j) = \omega_1 D(v_i, v_j) - \omega_2 D_{avg}(v_j) \]

- \( \omega_1, \omega_2 \) : cost function weights
- \( D(v_i, v_j) \) : shortest path between \( v_i \) and \( v_j \) according to graph \( G \)
- \( D(v_j) \) : node density around \( v_j \)

- Finds the path current node \( v_i \) to the next best target node \( v_j \)
  - visited nodes are remove from the node candidate list
  - shortest distance from current node
  - highest node density
Classification Strategy

- Classification confidence:
  
  \[ cc = \log \frac{P(Z_i = 'Treasure' \mid X_{i1} \ldots X_{ik})}{P(Z_i = 'NotTreasure' \mid X_{i1} \ldots X_{ik})} \]

- Classification decision conditions:
  - Sufficient confidence: \( cc > Th \)  
    Confidence threshold: \( Th \)
  - Insufficient balance: remained balance is less than the measurement cost
  - Time is up!
Immersive Satisficing Results
Approach

Experiment Setting
- Counting detected targets and classified targets in a fixed amount of time
- Compare the proposed path planning method with the shortest path planning method
- Vary the target configurations

<table>
<thead>
<tr>
<th>Performance Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D(τ)</strong></td>
</tr>
<tr>
<td><strong>V(τ)/D(τ)</strong></td>
</tr>
<tr>
<td><strong>R_y(τ)</strong></td>
</tr>
<tr>
<td><strong>η_y(τ)</strong></td>
</tr>
<tr>
<td><strong>η_v(τ)</strong></td>
</tr>
</tbody>
</table>
Results

**Trial #1 Cluster of targets in one region**

In this setup, the proposed robot planner heads to the region of denser information value first (left), while the shortest path planner heads to the isolated target (right). Although the shortest path planner covers more distance, it is less efficient.

<table>
<thead>
<tr>
<th>Performance Parameters</th>
<th>Proposed Planner</th>
<th>Shortest Path</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(τ)</td>
<td>19.51</td>
<td>25.07</td>
</tr>
<tr>
<td>V(τ)/D(τ)</td>
<td>0.4613</td>
<td>0.2792</td>
</tr>
<tr>
<td>η(τ)</td>
<td>0.2050</td>
<td>0.1197</td>
</tr>
<tr>
<td>R(τ)</td>
<td>1.00</td>
<td>0.75</td>
</tr>
<tr>
<td>ηv(τ)</td>
<td>0.4444</td>
<td>0.4286</td>
</tr>
</tbody>
</table>
Results

**Trial #2 Small number of scattered targets**

In this setup, there are no significant differences in the density of targets. Therefore, both planners give the same paths and have the same performance.

<table>
<thead>
<tr>
<th>Performance Parameters</th>
<th>Proposed Planner</th>
<th>Shortest Path</th>
</tr>
</thead>
<tbody>
<tr>
<td>D((\tau))</td>
<td>35.38</td>
<td>35.38</td>
</tr>
<tr>
<td>V((\tau))/D((\tau))</td>
<td>0.3675</td>
<td>0.3675</td>
</tr>
<tr>
<td>(\eta_y(\tau))</td>
<td>0.1696</td>
<td>0.1696</td>
</tr>
<tr>
<td>R((\tau))</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>(\eta_v(\tau))</td>
<td>0.4615</td>
<td>0.4615</td>
</tr>
</tbody>
</table>
Movie Demo
Summary (Year 2) and Future Work

- Biophysical modeling of satisficing strategies for inferential decision making
- Graphical model (BN) problem formulation
- AEA BN learning algorithm for satisficing, inferential decision making
- Time pressure / limited data / incomplete probabilistic model
- Semantic cues
- Integrate (passive satisficing) behavioral studies with:
  - Human (fMRI) neuroimaging
  - Non-human primates (NHPs) neural recordings
- Immersive environment and Webot-based software

Future Work:
- Active and dynamic satisficing task problem formulation
- Human and NHP behavioral studies (Webot - MATLAB® - DiVE)
- Mathematical modeling of active and dynamic satisficing control strategies
- Comparative studies and virtual experiments
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Questions?

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Backup Slides
Satisficing Decision and Control

Epistemic Utility Function [Curtis, Beard, Goodrich, Stirling, et al.]
- Seeks a family of control laws via model predictive control (or CLFs)
- Selectability criterion: control benefit (tracking, regulation, or stability)
- Rejectability criterion: control cost (time or control effort)

Satisficing Decision Trees [Simon, Kadane, Dieckmann, et al.]
- Inferential decision making
- Cues are predictors of performance
- Search for Spanish treasures
  - Unknown number of treasures at $n$ sites, $s_1, \ldots, s_n$
  - Given probabilities, $p(s_i)$, and cost $\phi(s_i)$, $i = 1, \ldots, n$
  - Find any treasure at minimum cost
- AND/OR trees, ordering constraints
Treasure Hunt Problem [Cai, Ferrari, et al.]
For a given layout $W \subset \mathbb{R}^3$ with $r$ targets and $n$ obstacles and a given joint probability mass function $p(z, \xi, \lambda)$, find the obstacle-free path that minimizes the distance traveled by a robot $A$ between two configurations $q_0$ and $q_f$, and maximizes the information gain, $I(z \mid \xi, \lambda)$, for a sensor with field-of-view $S$, installed on $A$. 
Online crowdsourcing service

Anonymous online subjects complete web-based tasks for small sums of money

Advantages
- Data can be collected quickly
- Diverse demographics

Empirical validation through replication using classic psychology tasks

[Crump, McDonnell & Gureckis, 2013]