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LABORATORY FOR INTELLIGENT SYSTEMS AND CONTROLS

ON THE DUALITY OF ROBOT AND SENSOR PATH PLANNING

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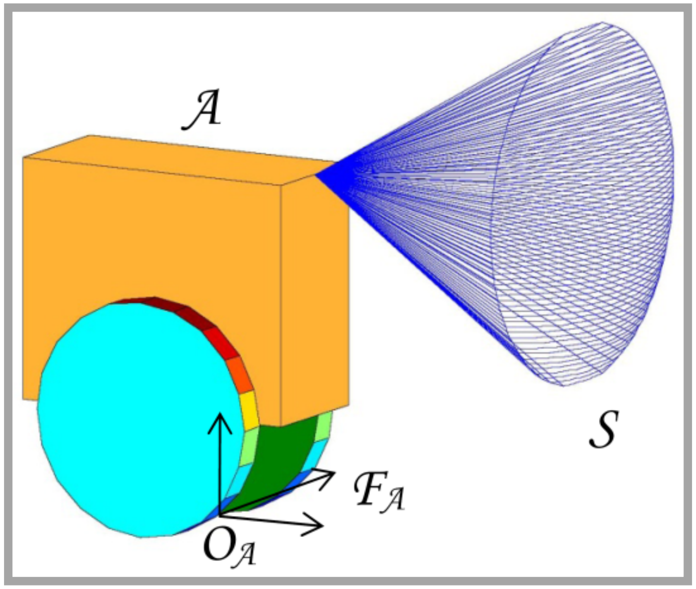
Introduction

- Motivation
  - Monitoring of urban environments
  - Localization of fugitive emissions
  - Monitoring of endangered species
  - Military reconnaissance missions
- Sensor Path Planning
  - Maximize the information profit
  - Minimize the performance costs
  - Optimize stochastic functions that are not necessarily additive
- Robot Path Planning
  - Concerned with purely navigational objectives
  - Optimize deterministic additive functions, such as the Euclidean distance



By formulating the sensor path planning problem as a mixed integer program (MIP) it can be shown to be a mathematical dual of the robot path planning problem

Literature Review

Artificial Potential Fields

- Well-established approach for robot path planning in the presence of obstacles
- Suited for online planning
- Subject to local minimia, and are often unable to account for the positions and geometries of targets as well as the FOV of the sensor
- Zhang and Ferrari (2009) proposed a novel approach to sensor path planning in which attractive potentials are generated for the targets, and by use of a potential function, a probabilistic roadmap is constructed, such that the sensor is able to escape local minimia
- Limited to local solutions, and are, therefore, unable to guarantee globally optimal sensor paths

Cell Decomposition

- Returns globally optimal solutions
- Cai and Ferrari (2009) adapted approximate cell decomposition in order to determine the optimal measurement strategy for a mobile sensor
- Obtaining an approximate cell decomposition of an environment can be computationally expensive and the techniques available cannot readily account for kinematic and dynamic motion constraints

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Literature Review

Mixed Integer Programming (MIP)

- Has been demonstrated to be a successful technique for determining collision free paths for robotic vehicles in obstacle populated environments (Richards et al. 2001, Blackmore and Williams 2006, Richards and How 2002)
- Although it can be computationally expensive, MIP is able to return globally optimal, complete solutions
- MIP has a flexible framework that allows for the consideration of a variety of planning constraints
    - Multi-agent safety guarantees (Schouwenaars et al. 2001, Schouwenaars et al. 2004, Pallottino et al. 2002)
    - Holonomic and nonholonomic kinematic constraints (Ma and Miller 2006, Swingler and Ferrari 2010, Swingler 2012)
    - Communication and control bounds (Bezzo et al. 2011)

This work develops a MIP approach for sensor path planning. Furthermore, by formulating the sensor path planning problem as a MIP it can be shown to be a mathematical dual of the robot path planning problem.

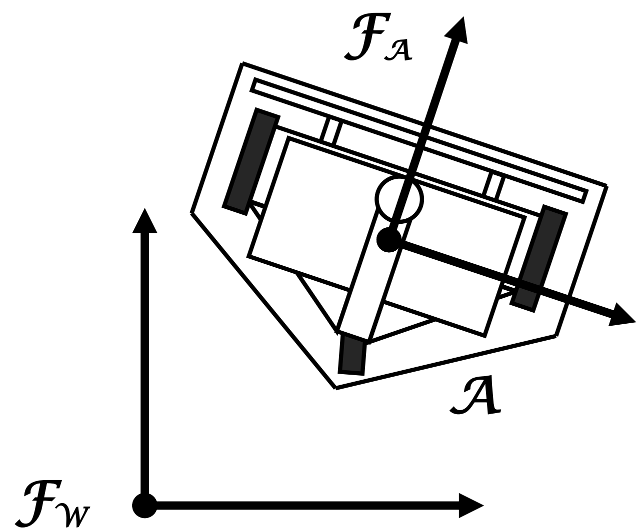
ROBOT PATH PLANNING

Problem Formulation and Assumptions

Determine a collision-free path of minimum cost for a mobile robot between two configurations in a region-of-interest, ROI.

The robot, $\mathcal{A} \subset \mathcal{W}$, consists of a closed and bounded subset of the workspace, $\mathcal{W} \subset R^N$

The workspace is populated by *n* fixed obstacles $\mathcal{B}_1, \ldots, \mathcal{B}_n$, where $\mathcal{B}_i \subset \mathcal{W}$, the positions and geometries of which are assumed known *a priori*



The robot configuration, $\mathbf{q} \in \mathcal{C}$, defines the position and orientation of a moving Cartesian reference frame $\mathcal{F}_\mathcal{A}$ embedded in $\mathcal{A}$ with respect to a fixed, Cartesian reference frame $\mathcal{F}_\mathcal{W}$ embedded in $\mathcal{W}$

The subset of the workspace occupied by the geometry of the robot with configuration $\mathbf{q}$ is denoted by $\mathcal{A}(\mathbf{q})$

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Problem Formulation and Assumptions

$$\min_{\mathbf{q},\mathbf{u}} J_c = \min_{\mathbf{q},\mathbf{u}} \int_0^\infty \mathcal{L}\left(\mathbf{q},\mathbf{u},t\right) \mathrm{d}t$$
$$\text{subject to} \quad \dot{\mathbf{q}}(t) = f\left(\mathbf{q},\mathbf{u},t\right)$$

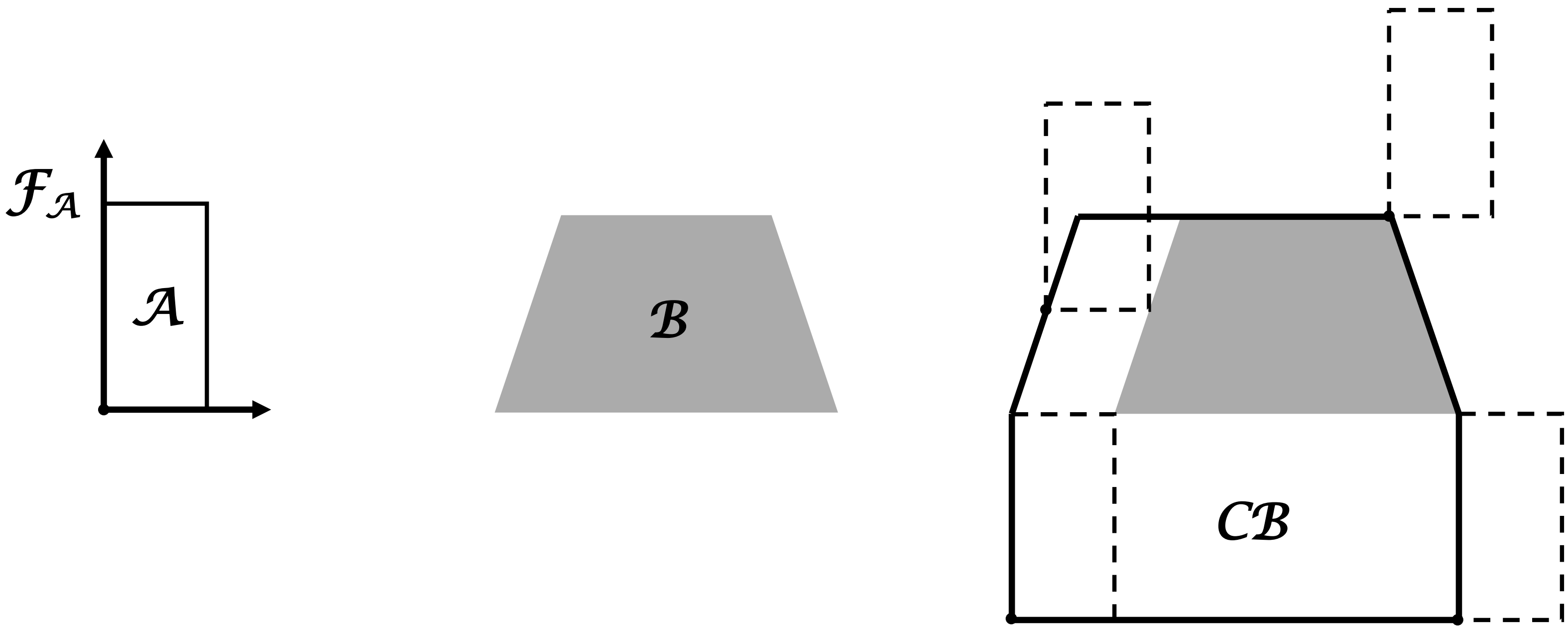
$$\min_{\mathbf{q},\mathbf{u}} J = \min \sum_{k=0}^{T} \left(\mathbf{q}^{\mathrm{T}}(k)\mathbf{P}\mathbf{q}(k) + \mathbf{u}^{\mathrm{T}}(k)\mathbf{R}\mathbf{u}(k)\right)$$
$$\text{subject to} \quad \forall k \in [0,\dots,T-1]$$
$$\mathbf{q}(k+1) = \mathbf{F}\mathbf{q}(k) + \mathbf{G}\mathbf{u}(k)$$
$$\mathbf{q}(k=0) = \mathbf{q}_0$$
$$\mathbf{q}(k=T) = \mathbf{q}_{\mathrm{F}}$$

C-Obstacles

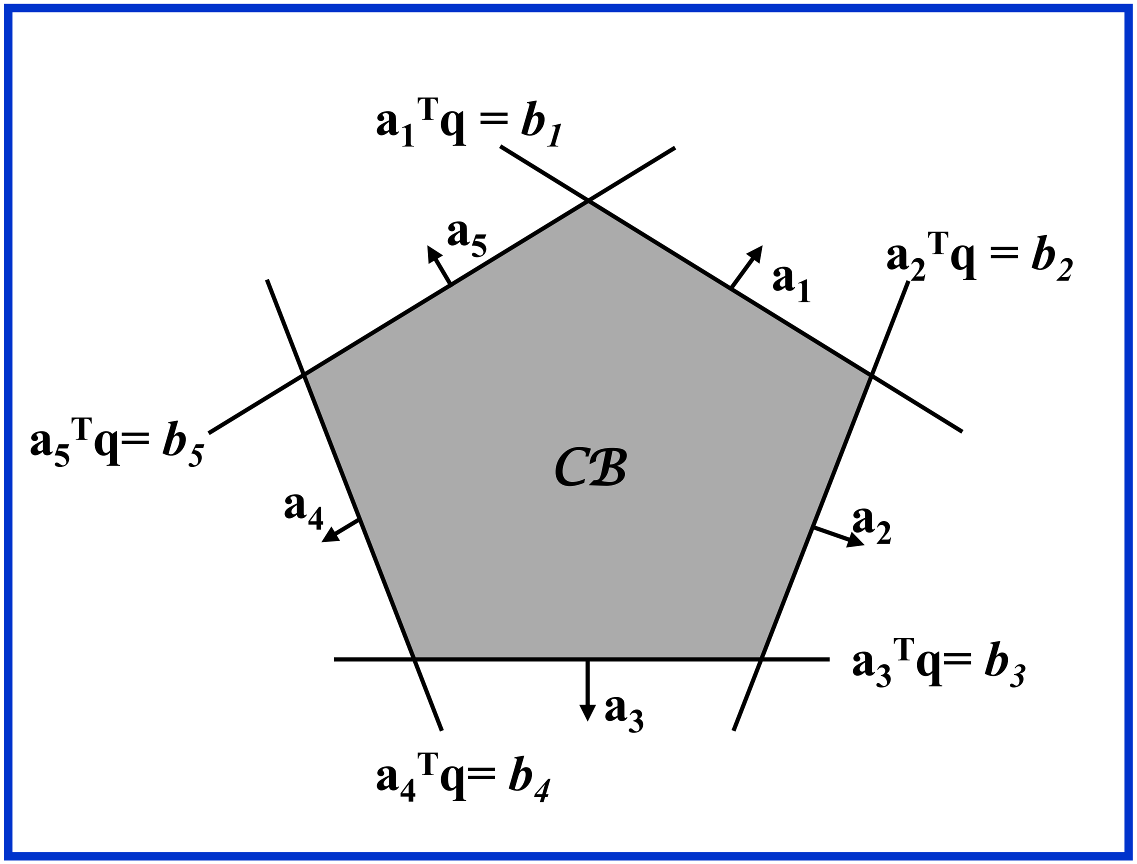
Introduce the concept of C-obstacles to account for the geometry of the robot

Let the C-obstacle $\mathcal{CB}_i$ represent a mapping of the obstacle $\mathcal{B}_i$ from the workspace into the configuration space

$$\mathcal{CB}_i \equiv \{\mathbf{q} \in \mathcal{C} | \mathcal{A}(\mathbf{q}) \cap \mathcal{B}_i \neq \emptyset\}$$



MIP for Robot Path Planning



$$\bigvee_{j=1,\ldots,s_i} \mathbf{a}_{ij}^{\mathrm{T}} \mathbf{q}(k) \geq b_{ij}$$

$$\mathbf{a}_{i1}^{\mathrm{T}} \mathbf{q}(k) \geq b_{i1} - M w_{i1}(k)$$
$$\mathbf{a}_{i2}^{\mathrm{T}} \mathbf{q}(k) \geq b_{i2} - M w_{i2}(k)$$
$$\vdots$$
$$\mathbf{a}_{is_i}^{\mathrm{T}} \mathbf{q}(k) \geq b_{is_i} - M w_{is_i}(k)$$
$$\sum_{j=1}^{s_i} w_{ij}(k) \leq (s_i - 1)$$

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MIP for Robot Path Planning

$$\mathbf{w}_i(k) = [w_{i1}(k)\ w_{i2}(k)\ \ldots\ w_{is_i}(k)]^T$$
$$\mathbf{A}_i = [\mathbf{a}_{i1}\ \mathbf{a}_{i2}\ \ldots\ \mathbf{a}_{is_i}]^T$$
$$\mathbf{b}_i = [b_{i1}\ b_{i2}\ \ldots\ b_{is_i}]^T$$

$$\mathbf{A}_i\mathbf{q}(k) + M\mathbf{w}_i(k) \geq \mathbf{b}_i$$
$$\boldsymbol{\alpha}_i^{\mathrm{T}}\mathbf{w}_i(k) \leq (s_i - 1)$$
$$w_{ij} = 0 \text{ or } 1 \quad \forall j \in [1, \ldots, s_i]$$

$$\min_{\mathbf{q},\mathbf{u}} J = \min \sum_{k=0}^{T} \left(\mathbf{q}^{\mathrm{T}}(k)\mathbf{P}\mathbf{q}(k) + \mathbf{u}^{\mathrm{T}}(k)\mathbf{R}\mathbf{u}(k)\right)$$

subject to $\forall k \in [0, \ldots, T-1]$

$$\mathbf{q}(k+1) = \mathbf{F}\mathbf{q}(k) + \mathbf{G}\mathbf{u}(k)$$
$$\mathbf{q}(k=0) = \mathbf{q}_0$$
$$\mathbf{q}(k=T) = \mathbf{q}_{\mathrm{F}}$$
$$\forall k \in [0, \ldots, T],\ \forall i \in [1, \ldots, n]$$
$$\mathbf{A}_i\mathbf{q}(k) + M\mathbf{w}_i(k) \geq \mathbf{b}_i$$
$$\boldsymbol{\alpha}_i^{\mathrm{T}}\mathbf{w}_i(k) \leq s_i - 1$$
$$w_{ij}(k) = 0 \text{ or } 1 \quad \forall j \in [1, \ldots, s_i]$$

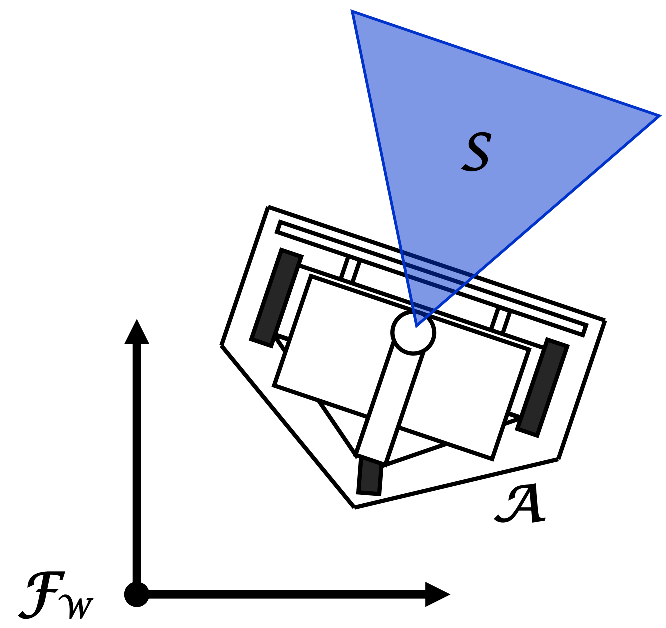
SENSOR PATH PLANNING

Problem Formulation and Assumptions

Determine a collision-free path of maximum information value between two configurations in a region-of-interest, ROI, or workspace $\mathcal{W}$.

The sensor is installed on a robotic platform with geometry $\mathcal{A} \subset \mathcal{W}$ such that its position an orientation of the field-of view, $\mathcal{S} \subset \mathcal{W}$, are fixed with respect to $\mathcal{A}$

The sensor is deployed in order to obtain measurements from *m* fixed targets $\mathcal{T}_1, \ldots, \mathcal{T}_m$, where $\mathcal{T}_i \subset \mathcal{W}$ the positions and geometries of which are assumed known *a priori*



The robot configuration, **q** ϵ $\mathcal{C}$, defines the position and orientation of $\mathcal{F}_\mathcal{A}$ in $\mathcal{F}_W$

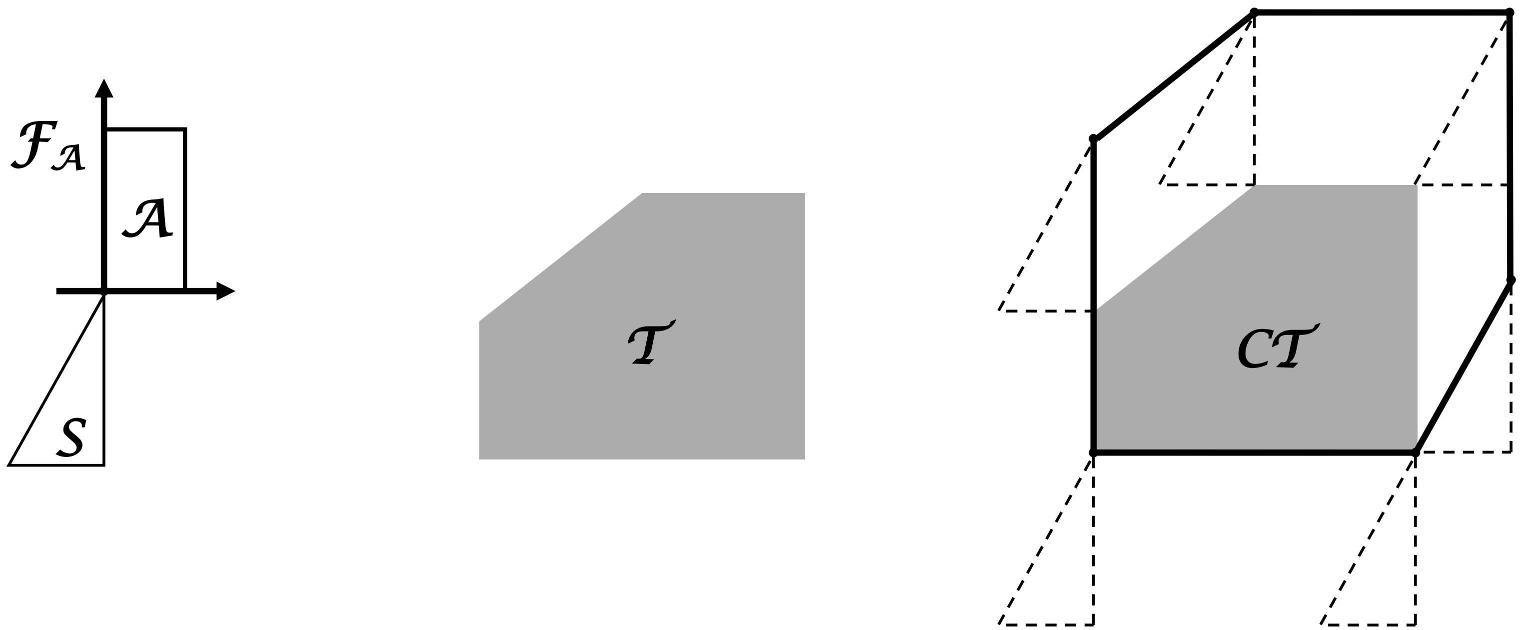
The subset of the workspace occupied by the FOV of the sensor with configuration **q** is denoted $\mathcal{S}$(**q**)

C-Targets

A target located at a point p in $\mathcal{S}(\mathbf{q})$ is measure by the sensor when the platform has configuration $\mathbf{q}$

A C-target $\mathcal{CT}_i$ is defined as a mapping of a target $\mathcal{T}_i$ from the workspace into the configuration space of the robotic sensor

$$\mathcal{CT}_i = \{\mathbf{q} \in \mathcal{C} | \mathcal{S}(\mathbf{q}) \cap \mathcal{T}_i \neq \emptyset\}$$



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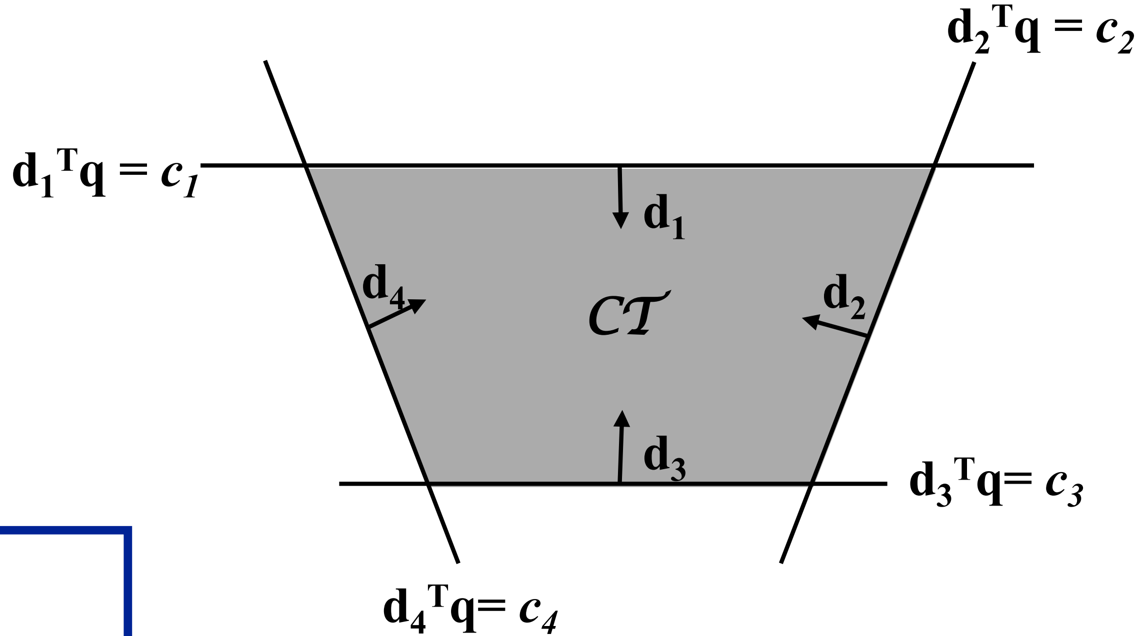
Duality of Robot and Sensor Planning

$$\mathbf{d}_{i1}^T \mathbf{q}(k) \leq c_{i1}$$
$$\mathbf{d}_{i2}^T \mathbf{q}(k) \leq c_{i2}$$
$$\vdots$$
$$\mathbf{d}_{ir_i}^T \mathbf{q}(k) \leq c_{ir_i}$$



$$\mathbf{D}_i \mathbf{q}(k) \leq \mathbf{c}_i$$

$$\mathbf{D}_i = [\mathbf{d}_{i1} \ \mathbf{d}_{i2} \ \ldots \ \mathbf{d}_{ir_i}]^T$$
$$\mathbf{c}_i = [c_{i1} \ c_{i2} \ \ldots \ c_{ir_i}]^T$$

$$\mathbf{D}_i \mathbf{q}(k) \leq \mathbf{c}_i + M z_i(k)$$
$$\sum_{k=0}^{T} z_i(k) \leq T$$
$$z_i = 0 \text{ or } 1$$

$$\mathbf{z}_i = [z_i(k=0) \ z_i(k=1) \ \ldots \ z_i(k=T)]^T$$

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Duality of Robot and Sensor Planning

$$\min_{\mathbf{q},\mathbf{u}} J = \min \sum_{k=0}^{T} \left(\mathbf{q}^{\mathrm{T}}(k)\mathbf{P}\mathbf{q}(k) + \mathbf{u}^{\mathrm{T}}(k)\mathbf{R}\mathbf{u}(k) \right)$$

$$\begin{aligned}
\text{subject to} \quad & \forall k \in [0, \ldots, T-1] \\
& \qquad \mathbf{q}(k+1) = \mathbf{F}\mathbf{q}(k) + \mathbf{G}\mathbf{u}(k) \\
& \mathbf{q}(k=0) = \mathbf{q}_0 \\
& \mathbf{q}(k=T) = \mathbf{q}_F \\
& \forall k \in [0, \ldots, T],\ \forall i \in [1, \ldots, m] \\
& \qquad -\mathbf{D}_i\mathbf{q}(k) + M z_i(k) \geq -\mathbf{c}_i \\
& \qquad \boldsymbol{\beta}_i^{\mathrm{T}}\mathbf{z}_i \leq T \\
& \qquad z_i(k) = 0 \text{ or } 1
\end{aligned}$$

Duality of Robot and Sensor Planning

$$\min_{\mathbf{q},\mathbf{u}} J = \min \sum_{k=0}^{T} (\mathbf{q}^{\mathrm{T}}(k)\mathbf{P}\mathbf{q}(k) + \mathbf{u}^{\mathrm{T}}(k)\mathbf{R}\mathbf{u}(k))$$

subject to $\forall k \in [0, \ldots, T-1]$

$$\mathbf{q}(k+1) = \mathbf{F}\mathbf{q}(k) + \mathbf{G}\mathbf{u}(k)$$

$$\mathbf{q}(k=0) = \mathbf{q}_0$$

$$\mathbf{q}(k=T) = \mathbf{q}_{\mathrm{F}}$$

$\forall k \in [0, \ldots, T],\ \forall i \in [1, \ldots, n]$

$$\mathbf{A}_i\mathbf{q}(k) + M\mathbf{w}_i(k) \geq \mathbf{b}_i$$

$$\boldsymbol{\alpha}_i^{\mathrm{T}}\mathbf{w}_i(k) \leq s_i - 1$$

$$w_{ij}(k) = 0 \text{ or } 1 \quad \forall j \in [1, \ldots, s_i]$$

Robot Path Planning Optimization Problem

$$\min_{\mathbf{q},\mathbf{u}} J = \min \sum_{k=0}^{T} (\mathbf{q}^{\mathrm{T}}(k)\mathbf{P}\mathbf{q}(k) + \mathbf{u}^{\mathrm{T}}(k)\mathbf{R}\mathbf{u}(k))$$

subject to $\forall k \in [0, \ldots, T-1]$

$$\mathbf{q}(k+1) = \mathbf{F}\mathbf{q}(k) + \mathbf{G}\mathbf{u}(k)$$

$$\mathbf{q}(k=0) = \mathbf{q}_0$$

$$\mathbf{q}(k=T) = \mathbf{q}_F$$

$\forall k \in [0, \ldots, T],\ \forall i \in [1, \ldots, m]$

$$-\mathbf{D}_i\mathbf{q}(k) + Mz_i(k) \geq -\mathbf{c}_i$$

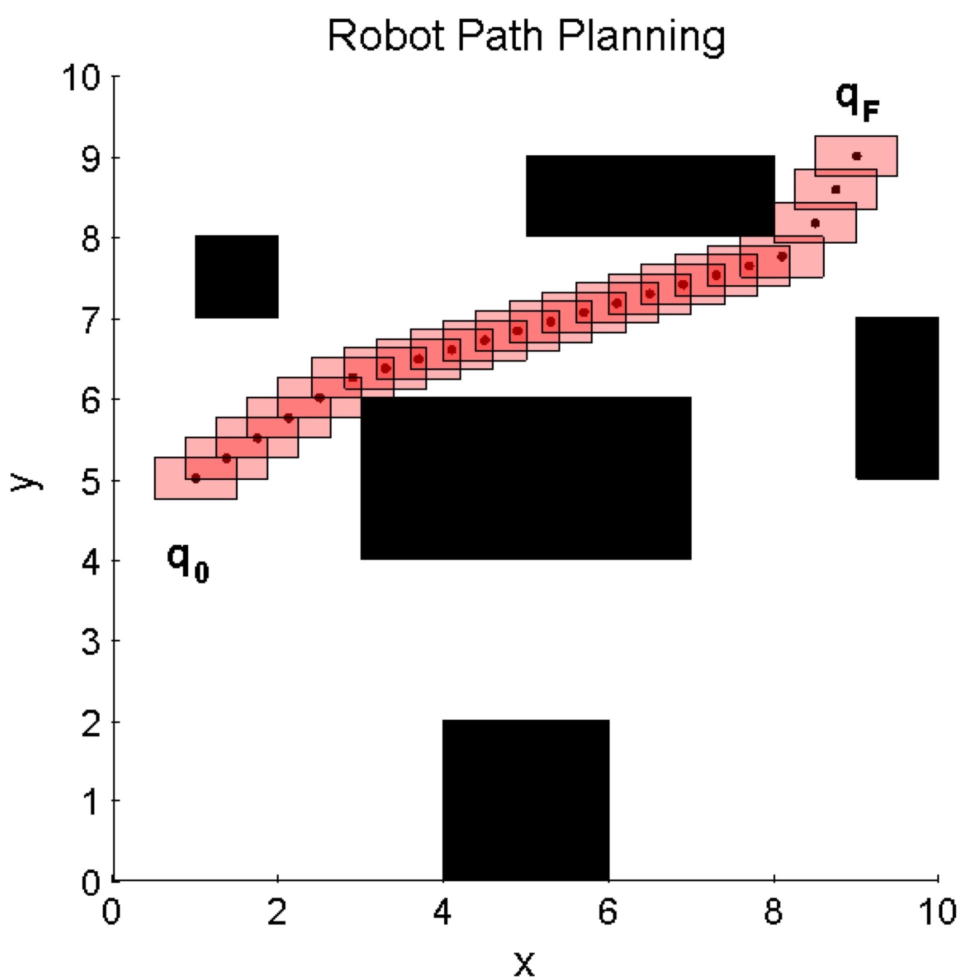
$$\boldsymbol{\beta}_i^{\mathrm{T}}\mathbf{z}_i \leq T$$

$$z_i(k) = 0 \text{ or } 1$$

Sensor Path Planning Optimization Problem

SIMULATIONS

Simulation Results: Robot



- Robot motion is restricted to translation
- Double integrator dynamics
- Minimize the L_2 distance traveled

Robot $\mathcal{A}$

Obstacles

Sensor

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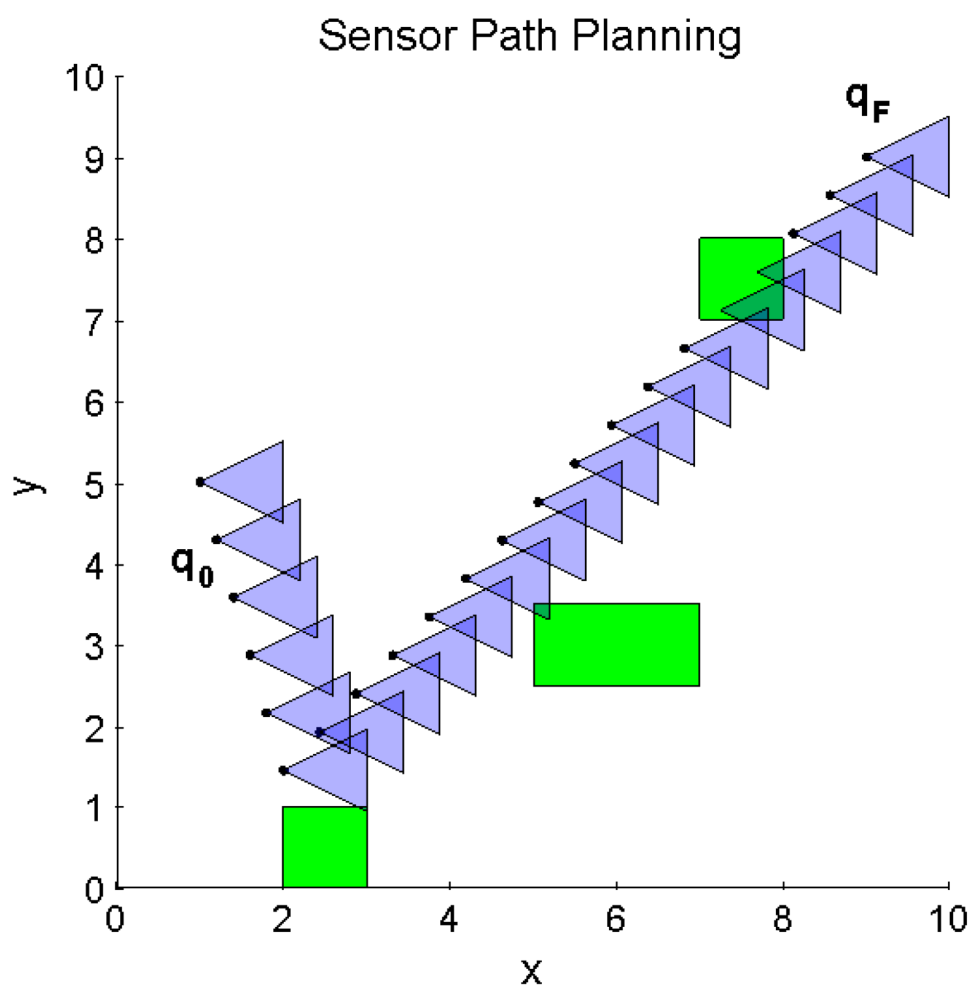
Simulation Results: Sensor



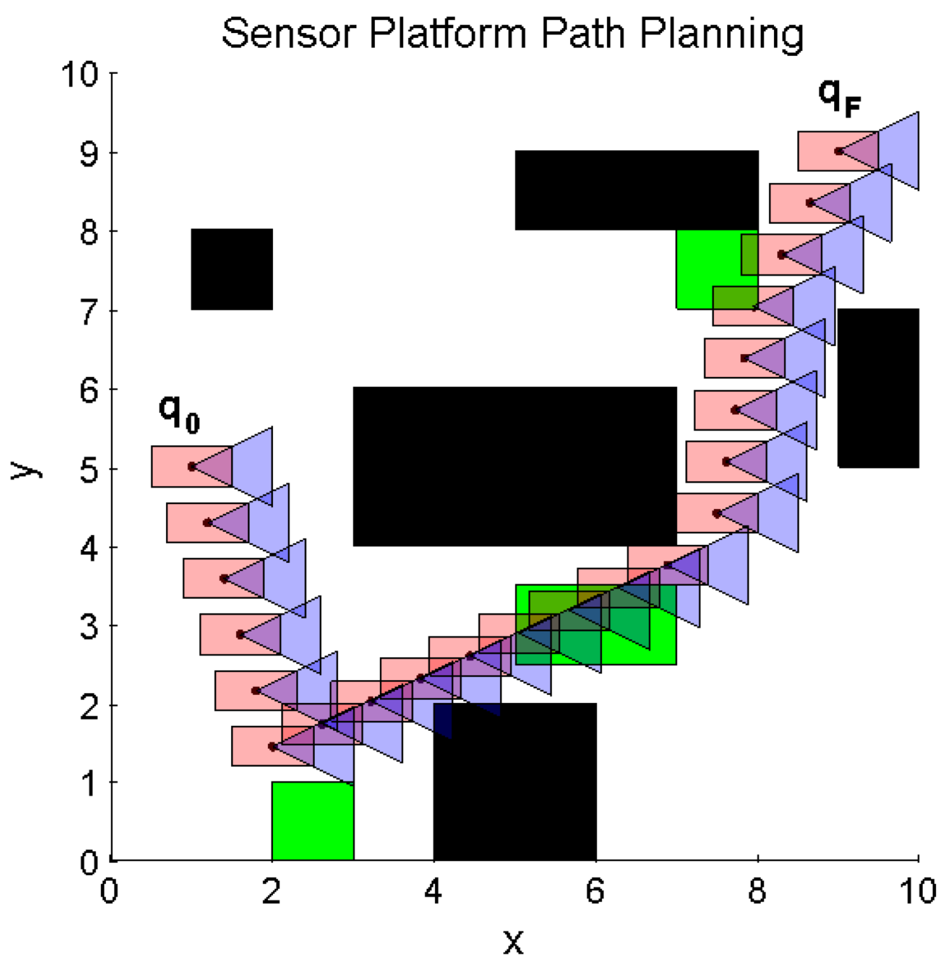
- Sensor is assumed to be a point mass with a bounded FOV
- Mobile sensor motion is restricted to translation
- Double integrator dynamics
- Minimize the L_2 distance traveled

Sensor FOV S

Targets

Sensor

Simulation Results: Robot



- Platform motion is restricted to translation
- Double integrator dynamics
- Minimize the L_2 distance traveled
- Account for the geometry of the robot and the FOV of the sensor

Robot $\mathcal{A}$

Obstacles

Sensor FOV $\mathcal{S}$

Sensor

Targets

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I would like to thank all of the members of the LISC for their hard work and support

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