



Sensor Planning for Multiple Targets Tracking

Hongchuan Wei

Advisor: Silvia Ferrari

Laboratory for Intelligent Systems and Controls

Cornell Robotics In Society Seminar

December 14, 2015

Outline

- Introduction and Motivation
- Problem Formulation
- Methodology: Information Based Sensor Planning
 - Target behavior described by single model
 - Target behavior described by mixture model mobile
 - Sensor dynamics constraints
- Simulation and Results
- Conclusion

Introduction and Motivation

Target behaviors learning

- Security surveillance
- Tracking endangered species
- Environmental monitoring

⋮



[1]



[2]



[3]



[1] <http://dowley.com/Services/VideoSurveillance/tabid/91/Default.aspx>
 [2] An Information Value Function For Nonparametric Gaussian Processes
 [3] www.h3c.com

Introduction and Motivation

- **Goal:** control actuated or configurable sensors to actively collect most valuable information
 - Estimator: compute the system state by fusing the data
 - **Planner:** determine the control by optimizing a function of costs and utilities
 - Actuator: follow the task execution as closely as possible
- **Challenge:**
 - Nonlinear target dynamics
 - Adaptive to data
 - Define 'value' of information
 - Number of sensors $<$ number of targets

Mathematical Problem Formulation

Target behaviors learning

- **Workspace:** $\mathcal{W} \subset \mathbb{R}^2$, convex polygon
- **Target dynamics:** unknown form

$$\dot{\mathbf{x}}_j(t) = \mathbf{f}[\mathbf{x}_j(t)], \quad j = 1, \dots, N$$

- **Sensor dynamics:** known

$$\dot{\mathbf{s}}(t) = \mathbf{g}[\mathbf{s}(t), \mathbf{u}(t)], \quad \mathbf{u}(t) \in U$$

- **Detection model:** limited sensor field of view (FOV)

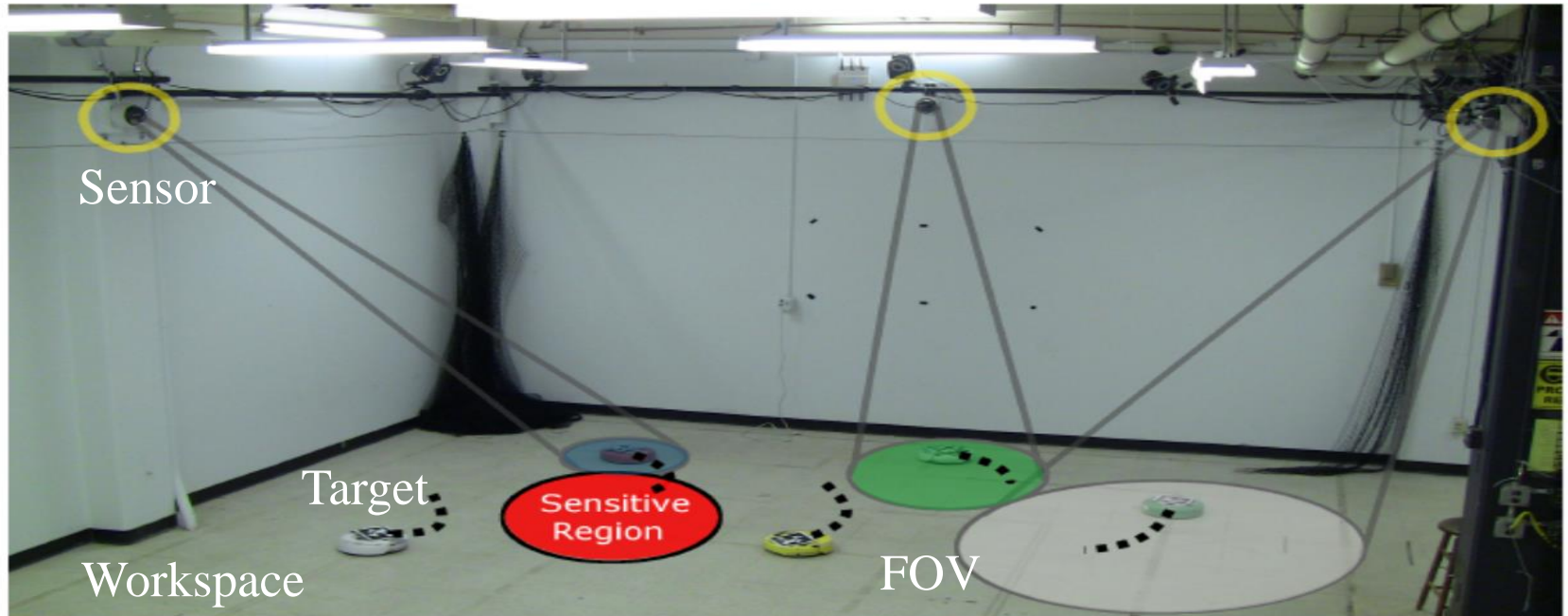
$$P_d = \begin{cases} 0 & : \mathbf{x}_j(t) \notin S(t) \\ 1 & : \mathbf{x}_j(t) \in S(t) \end{cases}$$

- **Measurement model:** known with additive noise

$$\mathbf{m}(t) = \mathbf{h}[\mathbf{x}(t), \mathbf{s}(t)] + \mathbf{n}, \quad \mathbf{n} \sim \mathcal{N}(0, \sigma^2)$$

- **Goal:** determine optimal control $\mathbf{u}^*(t)$ of sensors, such that collected measurements are most useful for learning \mathbf{f}

Example Problem



- **Workspace:** MIT raven testbed
- **Target dynamics:** unknown velocity field

$$\mathbf{f}: [x \ y]^T \rightarrow [\dot{x} \ \dot{y}]^T$$
- **Sensor dynamics:** linear with constraints
- **Detection model:** camera FOV
- **Measurement model:** $\mathbf{m}_j(t) = [\mathbf{x}_j(t) \ \mathbf{v}_j(t)] + \mathbf{n}$

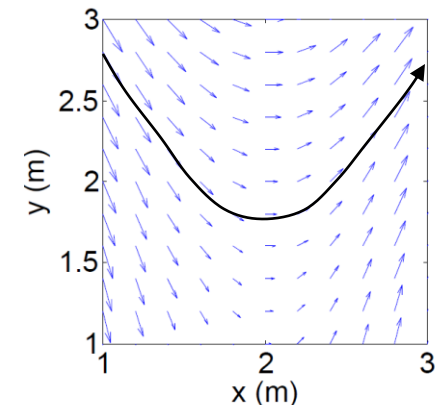


Fig. example of $\mathbf{f}[\mathbf{x}_j(t)]$



Methodology Part I: Target Dynamics Modelling

Methodology: Target Modeling

Single target: Gaussian process (GP) regression

$$\dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t)] \longrightarrow \{\mathbf{x}(t), \dot{\mathbf{x}}(t)\} \Rightarrow \mathbf{f}(\cdot)$$

A Gaussian process is a stochastic process, that is a collection of random variables $\{f(\mathbf{x}) \mid \mathbf{x} \in \mathcal{X}\}$ indexed by a set \mathcal{X} , for which any finite subset, $\{f(\mathbf{x}_1), \dots, f(\mathbf{x}_k)\}$, has a joint multivariate Gaussian distribution.

- Notation: $f(\mathbf{x}) \sim \text{GP}[\theta(\mathbf{x}), \phi(\mathbf{x}, \mathbf{x}')]]$
- Mean function: $\theta(\mathbf{x}) = \mathbb{E}_f[f(\mathbf{x})]$
- Covariance function: $\phi(\mathbf{x}, \mathbf{x}') = \mathbb{E}_f \{ [f(\mathbf{x}) - \theta(\mathbf{x})][f(\mathbf{x}') - \theta(\mathbf{x}')] \}^T$

- Covariance matrix: $\Phi(\mathbf{a}, \mathbf{b}) = \begin{bmatrix} \phi(\mathbf{a}_1, \mathbf{b}_1) & \cdots & \phi(\mathbf{a}_1, \mathbf{b}_n) \\ \vdots & \ddots & \vdots \\ \phi(\mathbf{a}_m, \mathbf{b}_1) & \cdots & \phi(\mathbf{a}_m, \mathbf{b}_n) \end{bmatrix}$

Gaussian Process Regression

Prediction on $\mathbf{f}(\boldsymbol{\xi}) = [f(\boldsymbol{\xi}_1) \ \cdots \ f(\boldsymbol{\xi}_k)]^T$ given data $\{\mathbf{p}, \mathbf{o}\}$:

$$\mathbf{f}(\boldsymbol{\xi}) \mid \{\mathbf{p}, \mathbf{o}\} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

- mean: $\boldsymbol{\mu} = \boldsymbol{\theta}(\boldsymbol{\xi}) + \boldsymbol{\Phi}(\boldsymbol{\xi}, \mathbf{p})\boldsymbol{\Phi}(\mathbf{p}, \mathbf{p})^{-1}[\mathbf{o} - \boldsymbol{\theta}(\mathbf{p})]$
- covariance: $\boldsymbol{\Sigma} = \boldsymbol{\Phi}(\boldsymbol{\xi}, \boldsymbol{\xi}) - \boldsymbol{\Phi}(\boldsymbol{\xi}, \mathbf{p})\boldsymbol{\Phi}(\mathbf{p}, \mathbf{p})^{-1}\boldsymbol{\Phi}(\mathbf{p}, \boldsymbol{\xi})$

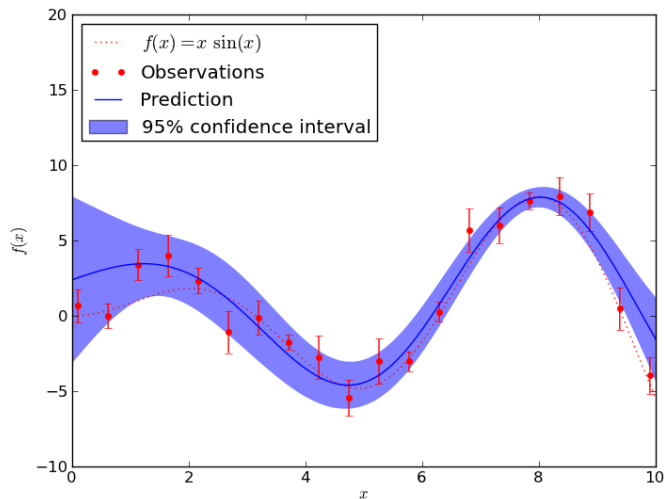


Fig. 1D GP regression example

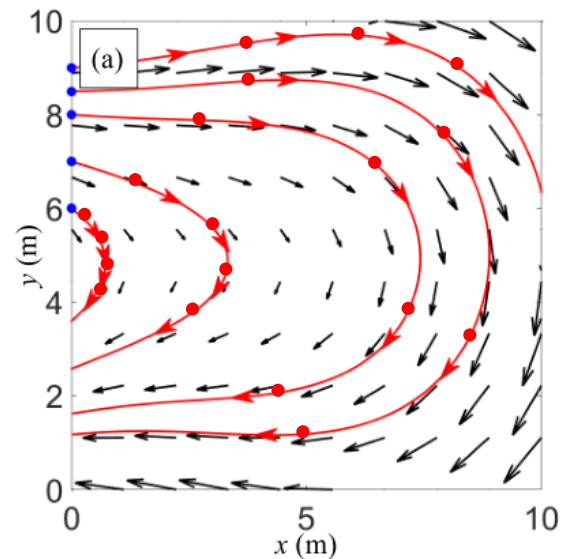


Fig. 2D GP regression example

Dirichlet Distribution

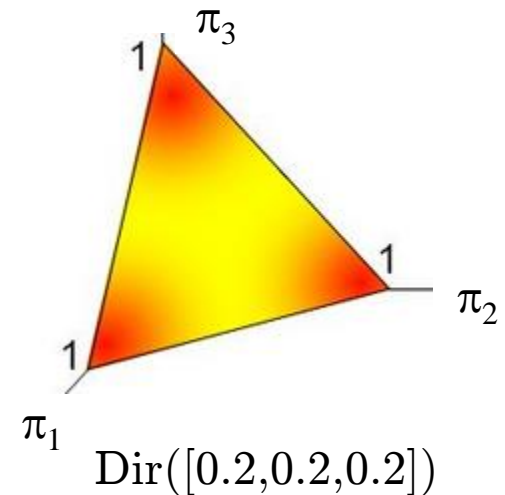
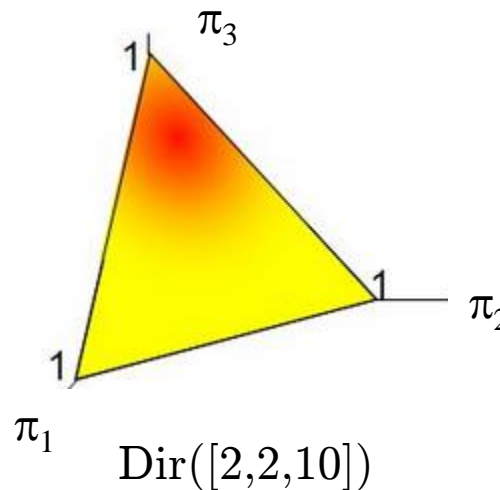
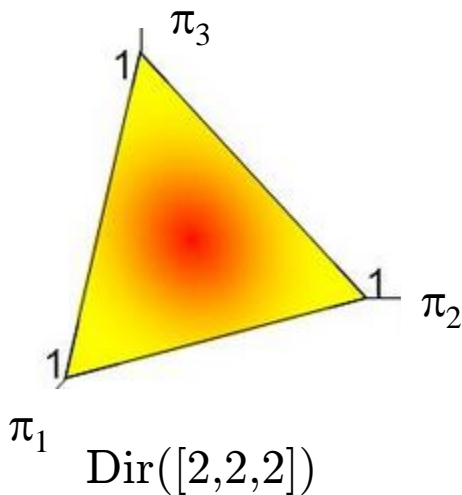
Dirichlet distribution: distribution over k -dimensional probability simplex

$$\boldsymbol{\pi} = [\pi_1 \ \cdots \ \pi_k]^T, \quad \text{for } \pi_k \geq 0, \quad \sum_k \pi_k = 1$$

- Density function of $\boldsymbol{\pi} \sim \text{Dir}([\alpha_1 \ \cdots \ \alpha_k])$

$$p(\boldsymbol{\pi}) = \frac{\Gamma(\sum_k \alpha_k)}{\prod_k \Gamma(\alpha_k)} \prod_k \pi_k^{\alpha_k - 1}$$

- Example

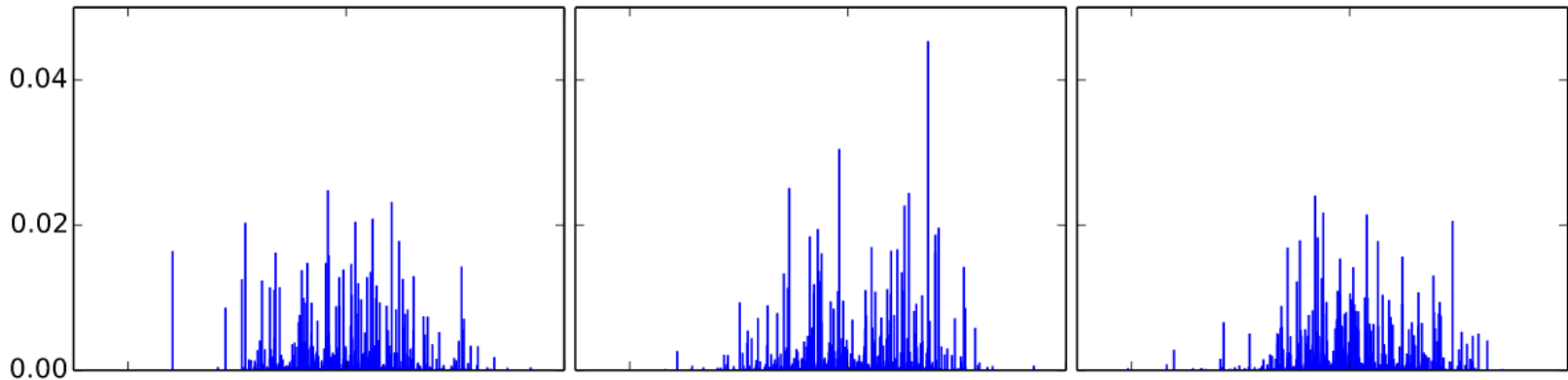


Dirichlet Process

Dirichlet process: distribution of infinite-dimension π

A Dirichlet process with parameters H and α , denoted by $DP[\alpha, H(A)]$, is a distribution of a random probability measure P , if for any finite measurable partition $\{B_i | 1 \leq i \leq n\}$ of A , it is true that

$$[P(B_1) \ \dots \ P(B_n)]^T \sim \text{Dir}([\alpha H(B_1) \ \dots \ \alpha H(B_n)])$$



- Three samples drawn from $DP(100, \mathcal{N}(0,0.1))$
- discrete distribution
- countably infinite number of point masses

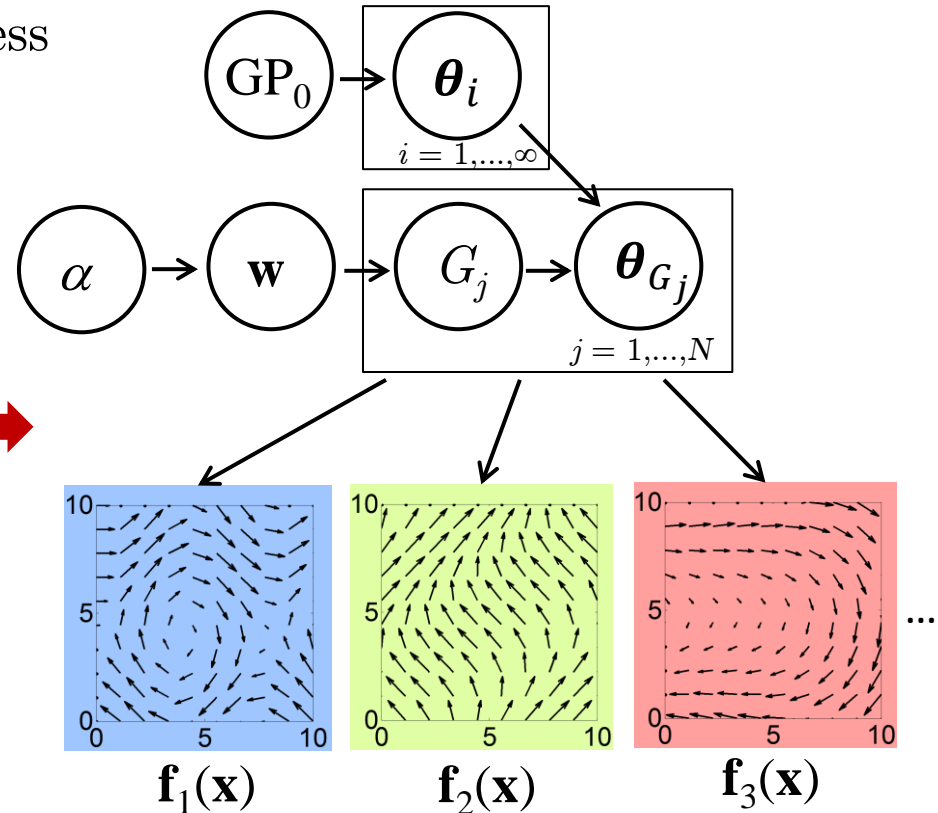
DPGP Mixture Model

Multiple targets: Dirichlet process prior over mixture of Gaussian process models (DPGP mixture model)

- Objective of DPGP-MM: describe target dynamics, $\{\mathcal{F}, \boldsymbol{\pi}\}$
 - Velocity field: 2D spatial phenomenon \leftarrow Gaussian process
 - Clustering \leftarrow Dirichlet process

- DPGP mixture model^[1]:

$$\begin{aligned} \{\boldsymbol{\theta}_i, \mathbf{w}\} &\sim \text{DP}(\alpha, \text{GP}_0), \quad i = 1, \dots, \infty \\ G_j &\sim \text{Cat}(\mathbf{w}), \quad j = 1, \dots, N \\ \mathbf{f}_{G_j}(\mathbf{x}) &\sim \text{GP}(\boldsymbol{\theta}_{G_j}, \boldsymbol{\Phi}), \quad j = 1, \dots, N \end{aligned}$$





Methodology Part II: Information Value of Measurement

DPGP Information Value

DPGP info-value: difference between **posterior** and **prior** DPGP model

$$\varphi[\mathbf{v}; \mathbf{m}(k+1)] = D\{p[\mathbf{v}|Q(k+1)]\|p[\mathbf{v}|Q(k)]\}$$

- D : Kullback-Leibler (or other) divergence

$$D(P_1 \| P_2) = \int_{-\infty}^{\infty} \ln \left(\frac{p_1(x)}{p_2(x)} \right) p_1(x) dx$$

- $\mathbf{v}_i = [\mathbf{f}_i(\xi_1)^T \quad \dots \quad \mathbf{f}_i(\xi_L)^T]^T$
- $\mathbf{v} = [\mathbf{v}_1^T \quad \dots \quad \mathbf{v}_M^T]^T$
- $Q(k) = \{\mathbf{m}(\ell), G(\ell) \mid 1 \leq \ell \leq k\}$
- velocity field-target association, G , is unknown
- future measurement, $\mathbf{m}(k+1)$, is unknown

Expected info-value: DPGP-EKLD

$$\hat{\varphi}[\mathbf{v}; \mathbf{m}(k+1)] = \mathbb{E}_{G_j} \left[\mathbb{E}_{\mathbf{m}(k+1)} [\varphi[\mathbf{v}; \mathbf{m}(k+1)]] \right]$$

- Assumptions:
 - VF-target association distribution learnt by DPGP model
 - measurement consistent with GP regression
- Theorem I: The GP-EKLD can be simplified as

$$\hat{\varphi}[\mathbf{v}; \mathbf{m}(k+1)] = \sum_{i,j} w_{ij} \int_{\mathcal{S}} h_i[\mathbf{x}_j(k+1)] \times p(\mathbf{x}_j(k+1)|Q(k)) d\mathbf{x}_j$$

where

$$h_i[\mathbf{x}_j(k+1)] = \frac{1}{2} \left[\text{tr} \left(\Sigma_{i,k}^{-1} \Sigma_{i,k+1} \right) - \ln \left(|\Sigma_{i,k+1} \Sigma_{i,k}^{-1}| \right) - 2L + \text{tr}(\mathbf{Q}^{-1} \mathbf{R}^T \Sigma_{i,k}^{-1} \mathbf{R} \mathbf{Q}^{-1}) \sigma_v^2 \right]$$

$$\mathbf{A} = \Phi[\mathbf{Y}_i(k), \mathbf{Y}_i(k)] + \sigma_v^2 \mathbf{I}_{2k}$$

$$\mathbf{R} = \Phi[\boldsymbol{\xi}, \mathbf{x}_j(k+1)] - \Phi[\boldsymbol{\xi}, \mathbf{Y}_i(k)] \mathbf{A}^{-1} \mathbf{B}$$

$$\mathbf{B} = \Phi[\mathbf{Y}_i(k), \mathbf{x}_j(k+1)]$$

$$\mathbf{Q} = \mathbf{D} - \mathbf{B}^T \mathbf{A}^{-1} \mathbf{B}$$

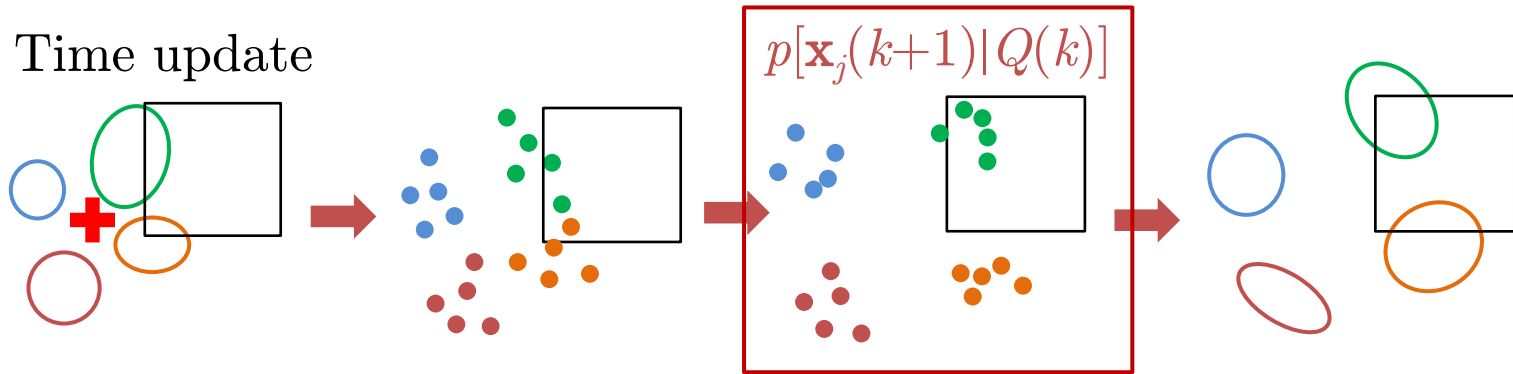
$$\mathbf{D} = \Phi[\mathbf{x}_j(k+1), \mathbf{x}_j(k+1)] + \sigma_v^2 \mathbf{I}_2$$

- $p[\mathbf{x}_j(k+1)|Q(k)]$ is hard to compute

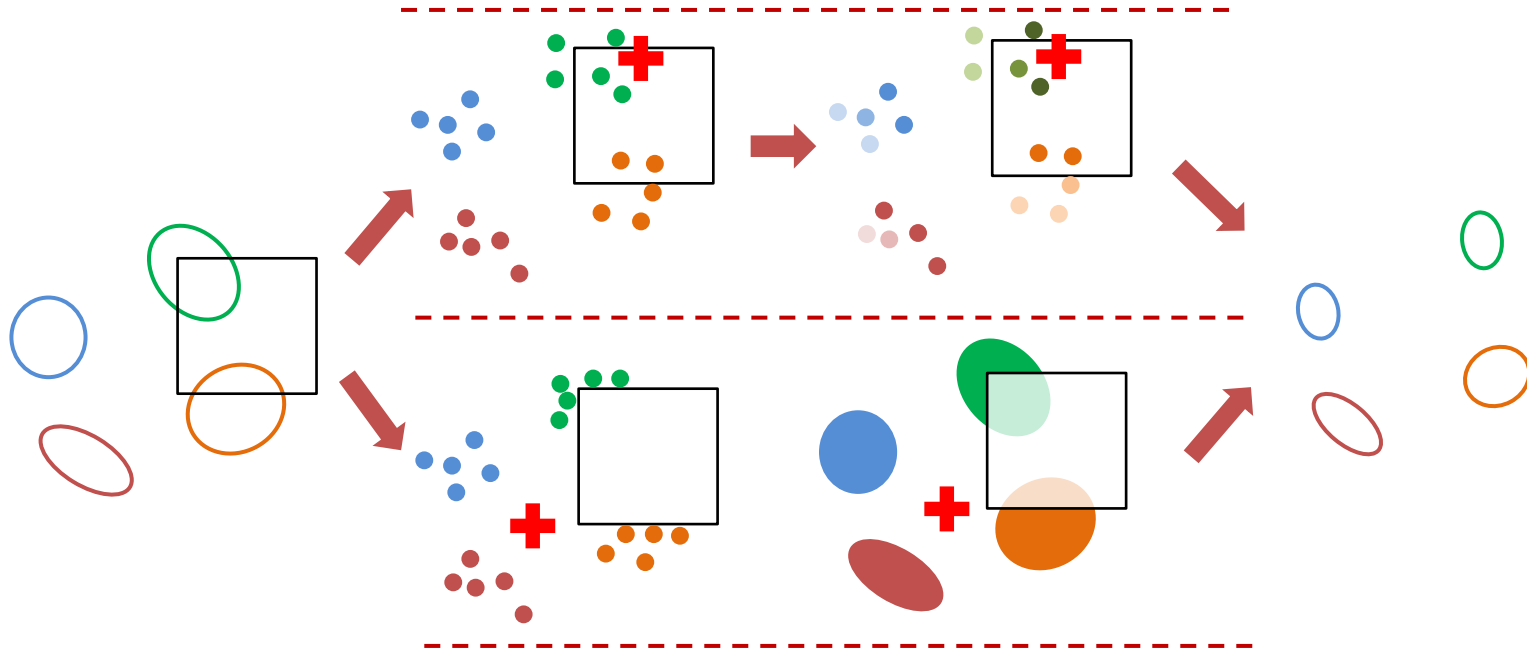
Gaussian Process Sum Particle Filter

Estimate $p[\mathbf{x}_j(k)|Q(k)] \sim \sum_{i=1}^M \mathcal{N}(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$

- Time update



- Measurement update



DPGP-EKLD (2)

Approximation of DPGP-EKLD

$$\hat{\varphi}[\mathbf{v}; \mathbf{m}(k+1)] \approx \sum_{i,j} \frac{w_{ij}}{S} \sum_{\mathbf{x}_j^{(\ell)} \in \mathcal{S}(k)} h_i[\mathbf{x}_j^{(\ell)}]$$

- Theorem II: The approximation is unbiased and variance of error decreases with sample number.

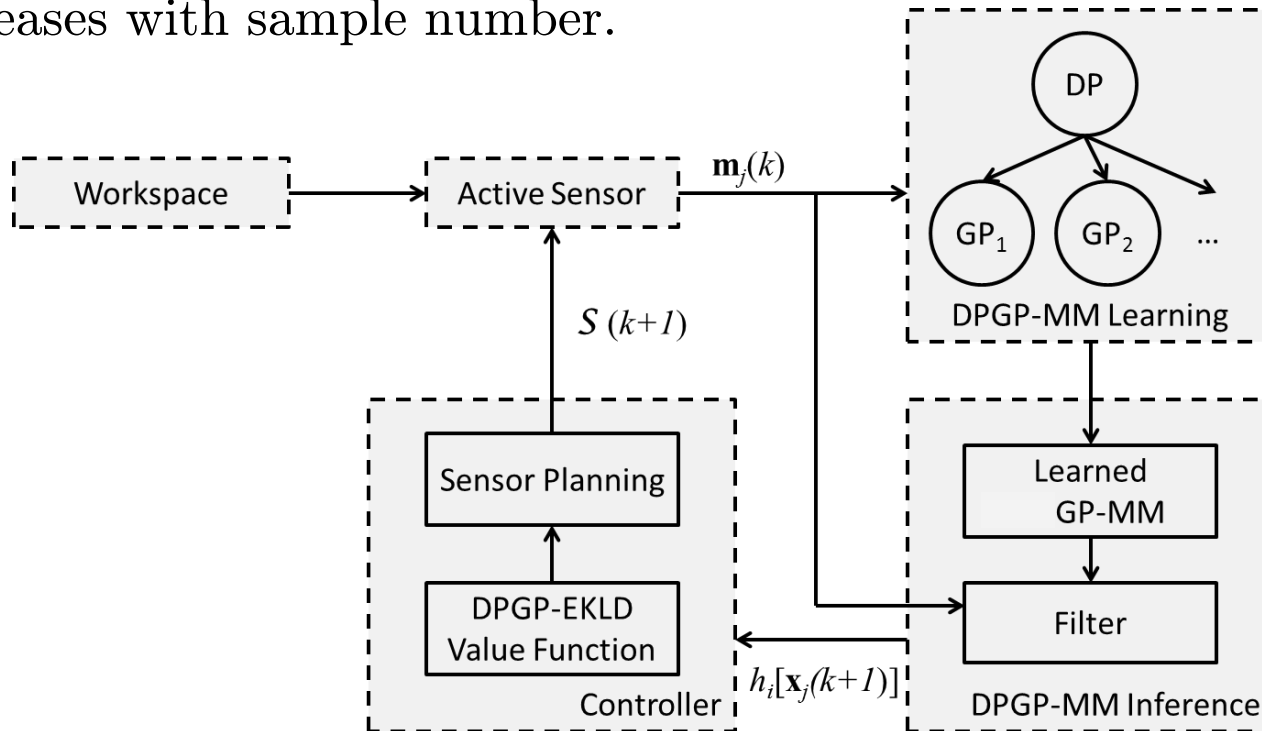


Fig. Framework of DPGP-EKLD sensor planning

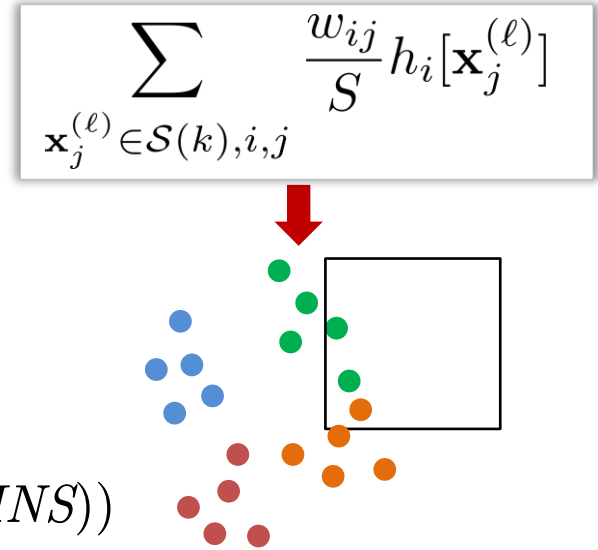
Greedy DPGP-EKLD Sensor Planning Algorithm

Examining DPGP-EKLD

- Free-flying sensor dynamics
- FOV shape does not change

Greedy algorithm to sensor planning

- Reduce planning to weighted points cover
- Each sample is weighted by $w_{ij}h_i[\mathbf{x}_j^{(\ell)}]/S$
- Complexity using segment tree: $O(MNS \log(MNS))$



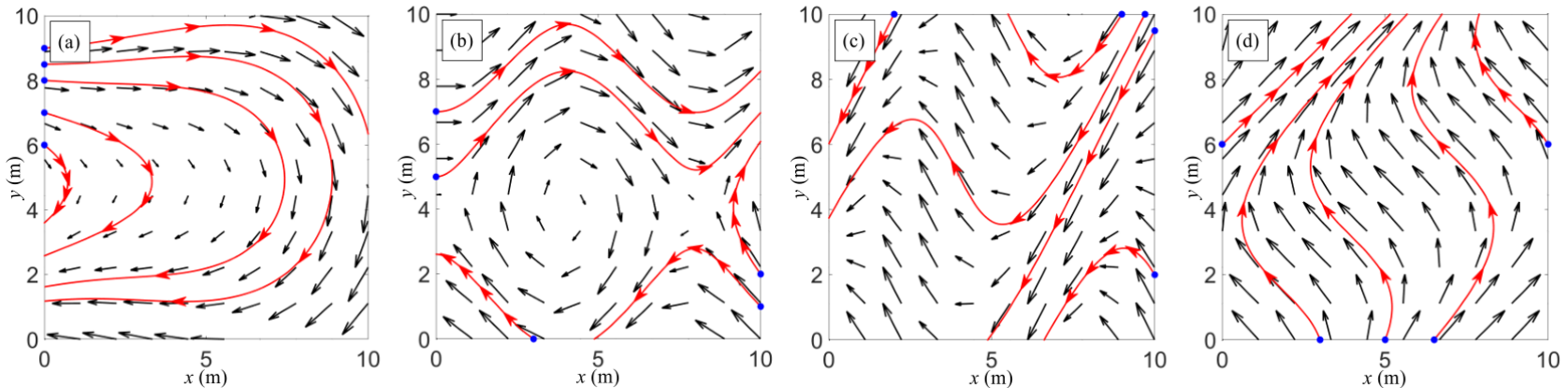
Algorithm 2 DPGP-EKLD

Require: $\theta(\cdot)$, $\phi(\cdot, \cdot)$, \mathcal{S} , ξ , N_f

- 1: **for** k **do** $= 1:N_f$
- 2: Sample target positions from the current target position distribution
- 3: Propagate the target positions to the next time step
- 4: Calculate the DPGP-EKLD for each propagated target position
- 5: Solve the weighted sum problem for every zoom level
- 6: Report the optimal FOV center position and the optimal zoom level
- 7: Carry out the control
- 8: **end for**

Example Problem for Examining DPGP-EKLD

- Workspace: $\mathcal{W} = \{x \in \mathbb{R}^2 \mid 1 \leq x \leq 10, 1 \leq y \leq 10\}$
- Sensor dynamics: free-flying objective
- Velocity fields: $\mathcal{F} = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4\}$



$$\mathbf{f}_1(\mathbf{x}) = \Phi(\mathbf{x}, \mathbf{p}_1) \mathbf{M}_1 \mathbf{O}_1$$

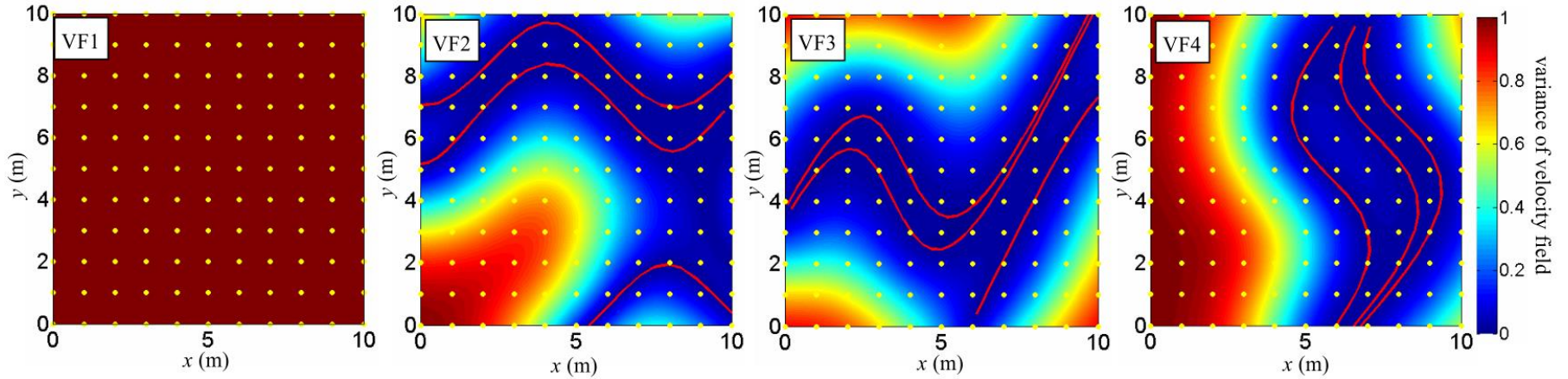
$$\mathbf{f}_2(\mathbf{x}) = \left[-\cos\left(\frac{\pi}{8}\mathbf{x}_{(2)}\right) \quad \sin\left(\frac{\pi}{4}\mathbf{x}_{(1)}\right) \right]^T$$

$$\mathbf{f}_3(\mathbf{x}) = [-0.5 \quad \sin([\pi/4 \quad 0.3]^T \mathbf{x})]^T$$

$$\mathbf{f}_4(\mathbf{x}) = [-\cos([\pi/8 \quad -0.5]^T \mathbf{x}) \quad 1]^T$$

- Probability of choosing every velocity field: $\boldsymbol{\pi} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}^T$

Result: Less Informative Prior



: Observed target trajectories

: Points of interest

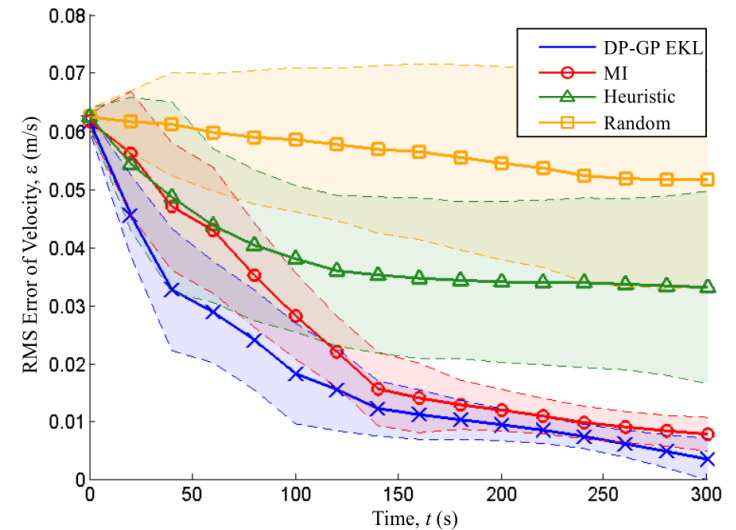
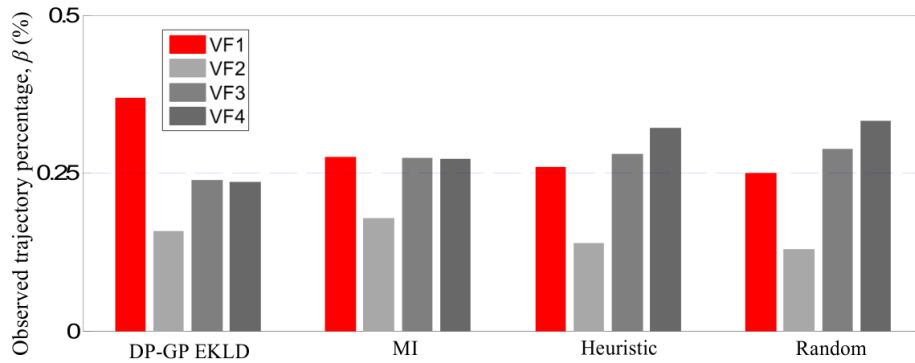


Fig. The distribution of observed trajectory percentage, averaged on the 50 runs of simulations

Fig. DPGP error



Methodology Part III: Incorporating Sensor Dynamics

Sensor Dynamics

- Linear sensor dynamics with constraints

$$\dot{\mathbf{s}}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{u}(t) \in U$$

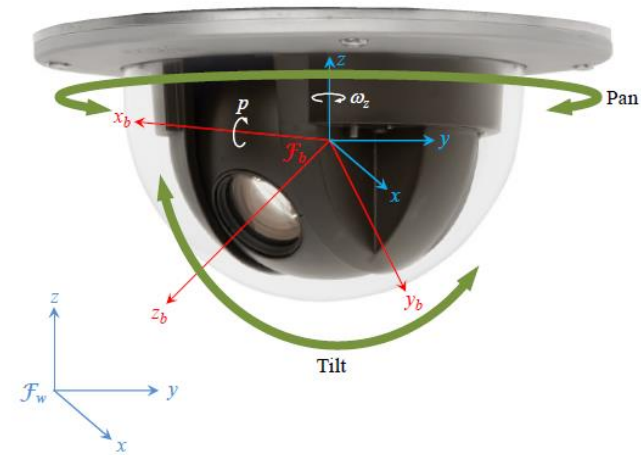
- Example: Pan-tilt (PT) Camera dynamics:

$$\mathbf{s} = [\psi \quad \Phi \quad \dot{\psi} \quad \dot{\Phi}]^T$$

$$\mathbf{u} = [u_1 \quad u_2]^T$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ k_1 & 0 \\ 0 & k_2 \end{bmatrix}$$

$$\mathbf{C}\mathbf{s} \leq \mathbf{1}_c \quad \text{and} \quad -\mathbf{1}_2 \leq \mathbf{u} \leq \mathbf{1}_2$$



- Equivalence to mutual information

$$\begin{aligned}
 \hat{\varphi}(\mathbf{v}; M_j(k, k')) &= \mathbb{E}_{M_j(k, k')} \left\{ \mathbb{E}_{G_j} \left\{ \varphi(\mathbf{v}; M(k, k')) \right\} \right\} \\
 &= \sum_{i=1}^M w_{ij} I(\mathbf{v}_i; M_j(k, k'))
 \end{aligned}$$

- Theorem III: Given rational number m and a rational covariance matrix $\mathbf{\Lambda}$ over a set of Gaussian random variables $V=S \cup U$, deciding whether there exists a subset $A \subset S$ of cardinality k such that $I(A; U) \geq m$ is *NP*-complete.
- **Theorem IV**: Finding the optimal control trajectory that maximizes the DPGP-EKLD, subject to the constraints on the camera state and the control input, is *NP*-hard.
 - Proof by restriction
 - Targets are static
 - Sensor moves fast enough as if free-flying

Lower Bound of DPGP-EKLD

- Theorem V: The DPGP-EKLD evaluated for measurements obtained between time step k and time step k' is lower bounded by discounted summation of the mutual information as follows,

$$\hat{\varphi}(\mathbf{v}; M_j(k, k')) \geq \sum_{i=1, \ell=k}^{M, k'} (1-\gamma)\gamma^{\ell-k} I(\mathbf{v}_i; \mathbf{m}_j(\ell))$$

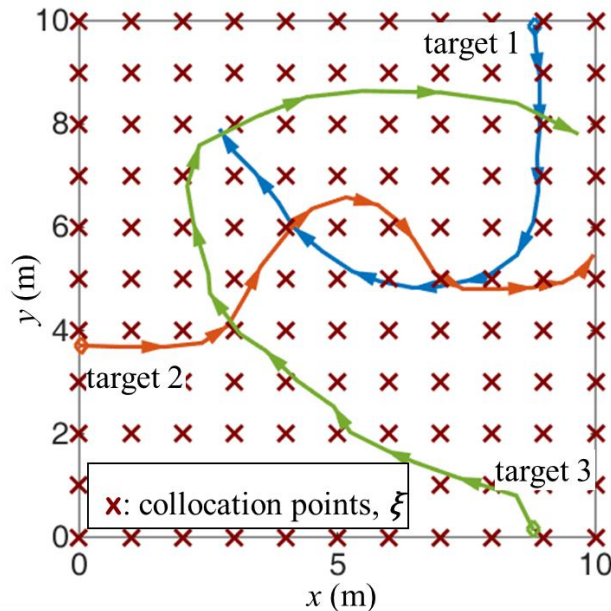


Fig. 1 Simulation scenario for validation of the additive lower bound.

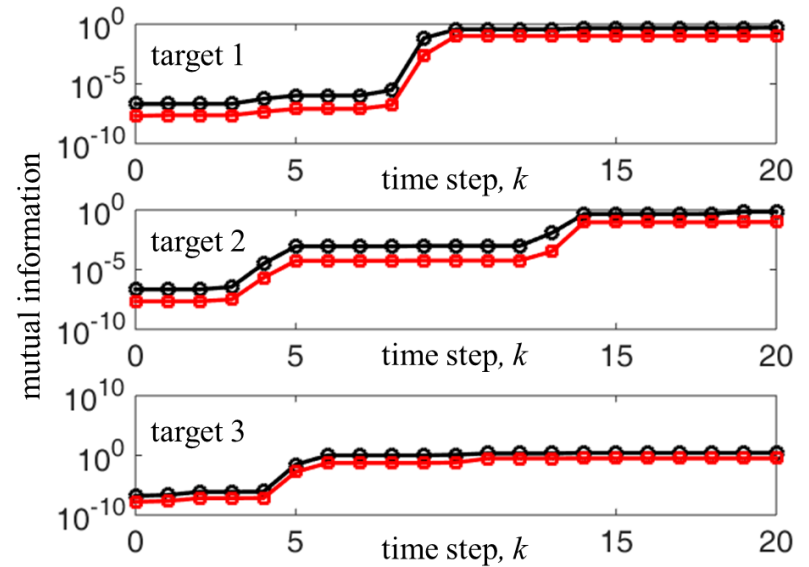


Fig. 2 DPGP-EKLD (black line) and the additive lower bound (red line)

Receding Horizon Control

- Objective function:

$$J_{ij} \triangleq w_{ij} \sum_{\ell=k}^{k'} (1-\gamma)\gamma^{\ell-k} I(\mathbf{v}_i; \mathbf{m}_j(\ell)) P(\mathbf{s}(\ell), \mathbf{x}_j(\ell))$$

$$P(\mathbf{s}, \mathbf{x}_j) = 1 - \|\mathbf{H}\mathbf{s} - \mathbf{h}(\mathbf{x}_j)\|^2/h$$

- Receding horizon control
 - closed-loop control strategy: target/sensor state update
 - constraints: at least finite horizon
 - infinite horizon solution obsolete after DPGP-MM update
 - lower computational complexity

$$\text{maximize}_{\mathbf{u}(\ell), k \leq \ell \leq k'} [J_{11} \quad \cdots \quad J_{MN}]^T$$

$$\text{subject to } \mathbf{s}(k) = \mathbf{s}_0$$

$$\mathbf{s}(\ell + 1) = \mathbf{A}\mathbf{s}(\ell) + \mathbf{B}\mathbf{u}(\ell), \ell = k, \dots, k'$$

$$-\mathbf{1} \leq \mathbf{u}(\ell) \leq \mathbf{1}, \ell = k, \dots, k'$$

Lexicographic Algorithm

Lexicographic algorithm for multiple output optimization

$$\begin{aligned}
 & \max_{\boldsymbol{\chi}} J'_i(\boldsymbol{\chi}) \\
 & \text{s.t. } J'_j(\boldsymbol{\chi}) \geq J_j^*, j = 1, \dots, i-1 \\
 & \boldsymbol{\chi} \triangleq [\mathbf{s}^T(k) \ \dots \ \mathbf{s}^T(k') \ \mathbf{u}^T(k) \ \dots \ \mathbf{u}^T(k')]^T \in U
 \end{aligned}$$

- For the same target, objective function corresponding to velocity field with higher target-VF association probability is more important

$$J_{ij} \succeq J_{i'j} \Leftrightarrow w_{ij} \geq w_{i'j}, \text{ for } i \neq i', j = 1, \dots, N$$

- For different targets, objective function corresponding to velocity field with higher target-VF association probability is also more important

$$J_{ij} \succeq J_{i'j'}, \text{ for } i < i', j \neq j'$$

- When $i=i'$, use ideal value to determine the sequence

$$J_{ij}^I \triangleq \max_{\boldsymbol{\chi}} \{J_{ij}(\boldsymbol{\chi}) \mid \boldsymbol{\chi} \in U\}$$

$$J_{ij} \succeq J_{ij'} \Leftrightarrow J_{ij}^I \geq J_{ij'}^I, \text{ for } i = 1, \dots, M, j \neq j'$$

Lexicographic Algorithm

- First iteration: convex quadratic programming with linear constraints

$$J_{ij} = \left(\sum_{\ell=k}^{k'} \beta(\ell) - \mathbf{c}^T \mathbf{c} \right) - \left(\mathbf{x}^T \mathbf{Q}^T \mathbf{Q} \mathbf{x} - \mathbf{c}^T \mathbf{Q} \mathbf{x} \right)$$

where,

$$\mathbf{Q} \triangleq \begin{bmatrix} \sqrt{\frac{\beta(k)}{h}} \mathbf{H} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \sqrt{\frac{\beta(k+1)}{h}} \mathbf{H} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \sqrt{\frac{\beta(k')}{h}} \mathbf{H} \end{bmatrix} \mathbf{0}_{2K \times 2K}$$

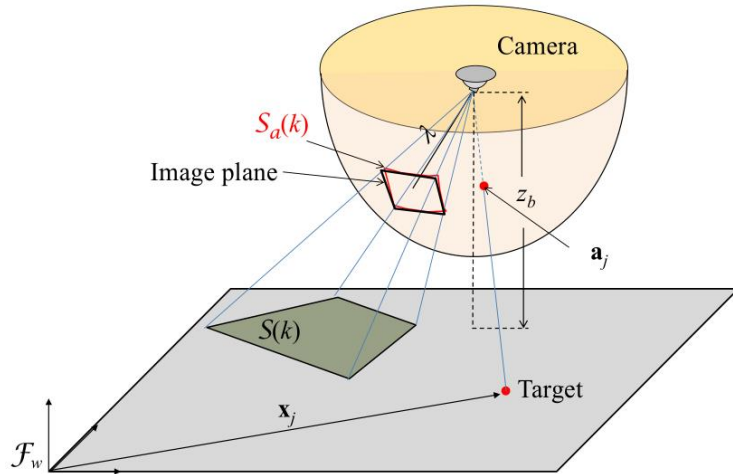
$$\mathbf{c} \triangleq \left[\sqrt{\frac{\beta(k)}{h}} \mathbf{h}^T [\mathbf{x}_j(k)] \cdots \sqrt{\frac{\beta(k')}{h}} \mathbf{h}^T [\mathbf{x}_j(k')] \right]^T$$

$$\beta(\ell) \triangleq w_{ij} (1 - \gamma) \gamma^{\ell-k} I(\mathbf{v}_i; \mathbf{m}_j(\ell)), \quad \ell = k, \dots, k'$$

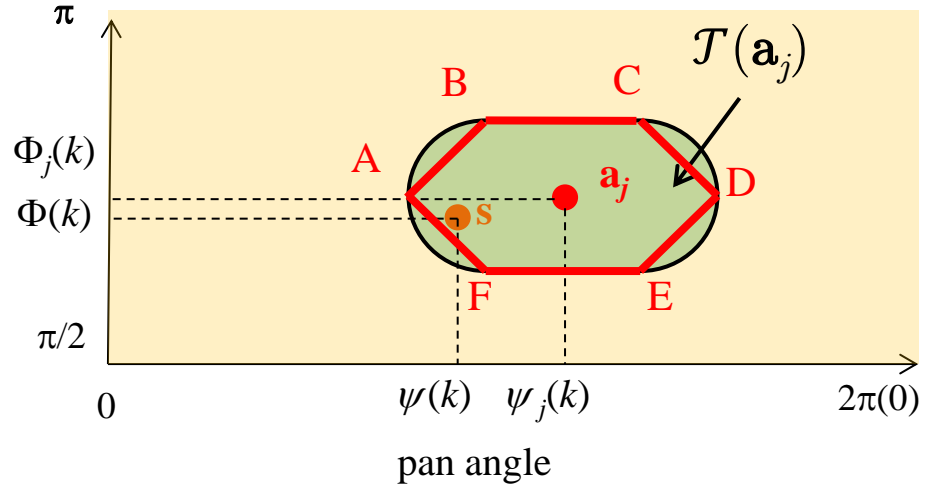
Lexicographic Algorithm

- The i th iterations: additional constraints

$$J'_j(\boldsymbol{\chi}) \geq J'_j^*, \quad j = 1, \dots, i - 1$$



tilt angle



- $\mathbf{a}_j \in S_a(k) \Leftrightarrow \mathbf{s} \in \mathcal{T}(\mathbf{a}_j)$
- $\mathcal{T}(\mathbf{a}_j) : |\Delta\Phi| \leq \tan^{-1}[h/(2\lambda)], \quad |\Delta\psi| \leq \tan^{-1}\left(\frac{w}{2\lambda} \sec(\Phi_j) \cos(\Delta\Phi)\right)$
- $\mathcal{T}(\mathbf{a}_j)$ is convex, symmetric, horizontal upper and lower bound
- Theorem VI: Area of polygon $ABCDEF$ divided by area of $\mathcal{T}(\mathbf{a}_j)$ is lower bound by $1 - (\sqrt{2} - 1)^2/2 \approx 91.4\%$, if the view angle of camera is no more than 90° .

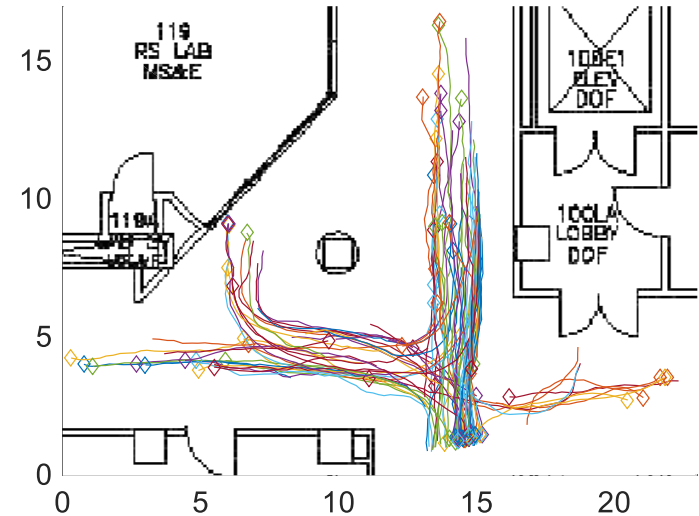


Simulations and Results

Simulation Setup

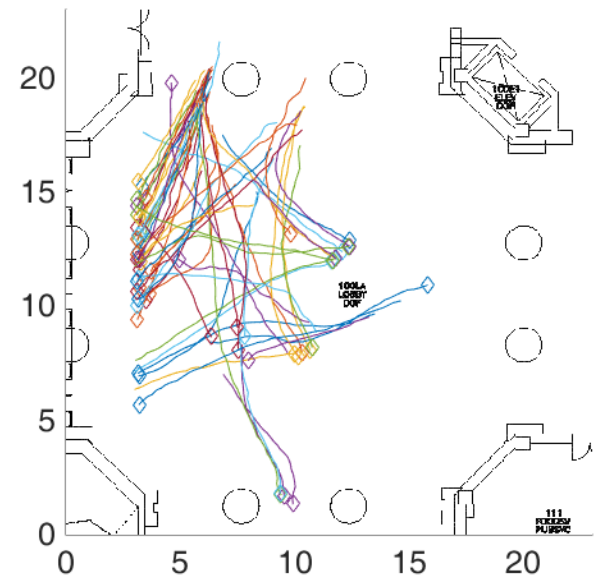
Data set 1 (thanks to Miao Liu@MIT)

- Pedestrian data set collected at Building 4, MIT
- 91 targets: 50 training, 41 testing
- Sampling time: 0.1s
- Subsampled to remove stops
- Workspace: 23m x 17 m



Data set 2 (thanks to Miao Liu@MIT)

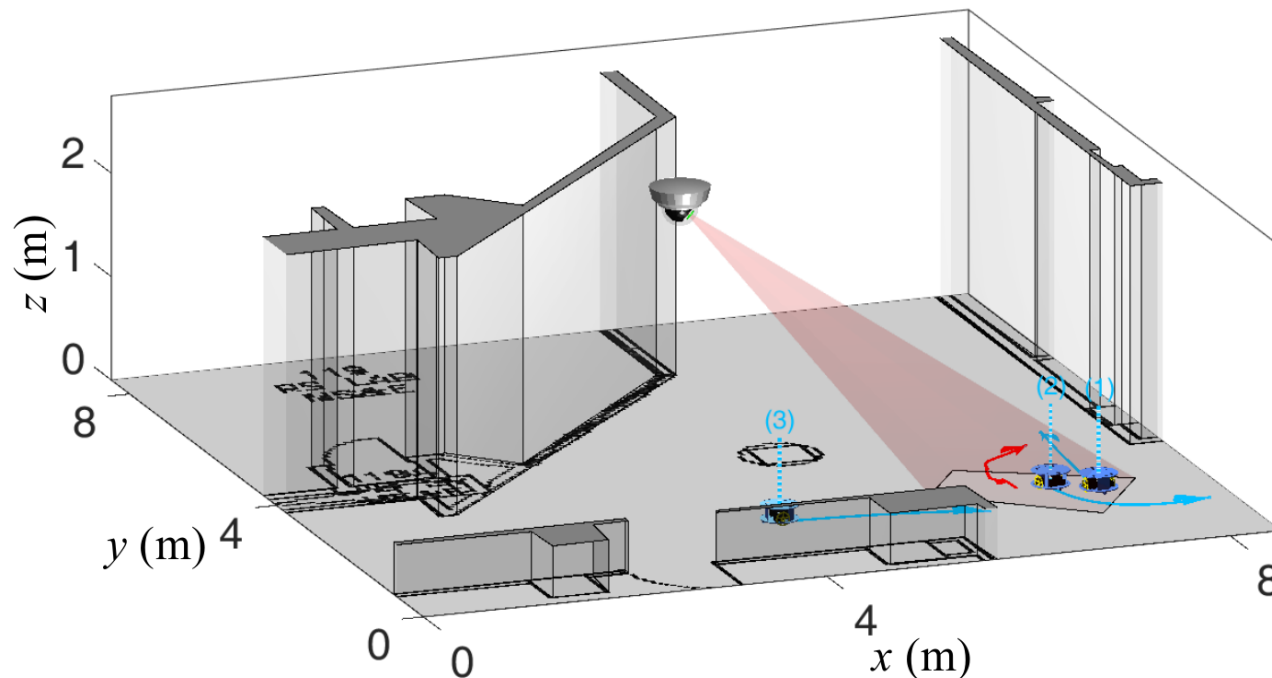
- Pedestrian data set collected at Hallway
- 73 targets: 50 training, 23 testing
- Sampling time: 0.1s
- Subsampled to remove stops
- Workspace: 20m x 20 m



Simulation Setup

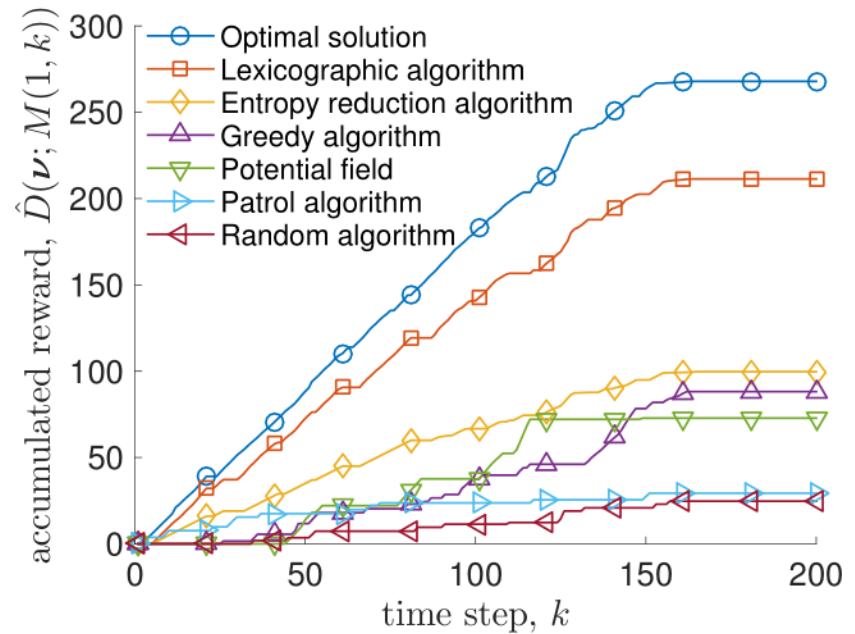
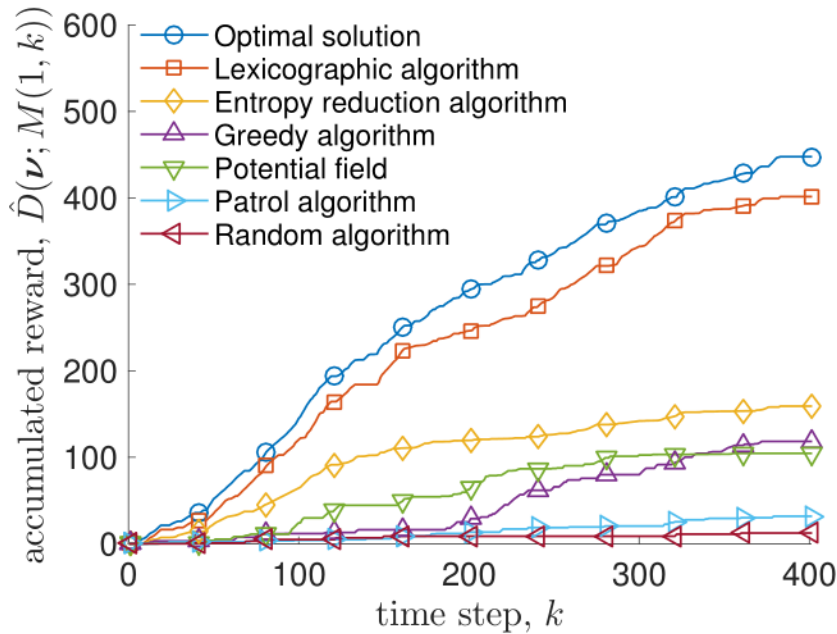
Sensor (simulated)

- AXIS P5624-E PTZ Dome Network Camera
- Pan: 360 degree endless, 0.2 - 350 degree/second
- Tilt: 180 degree, 0.2 - 350 degree/second
- Positioned in the center of hall way



Simulation Result

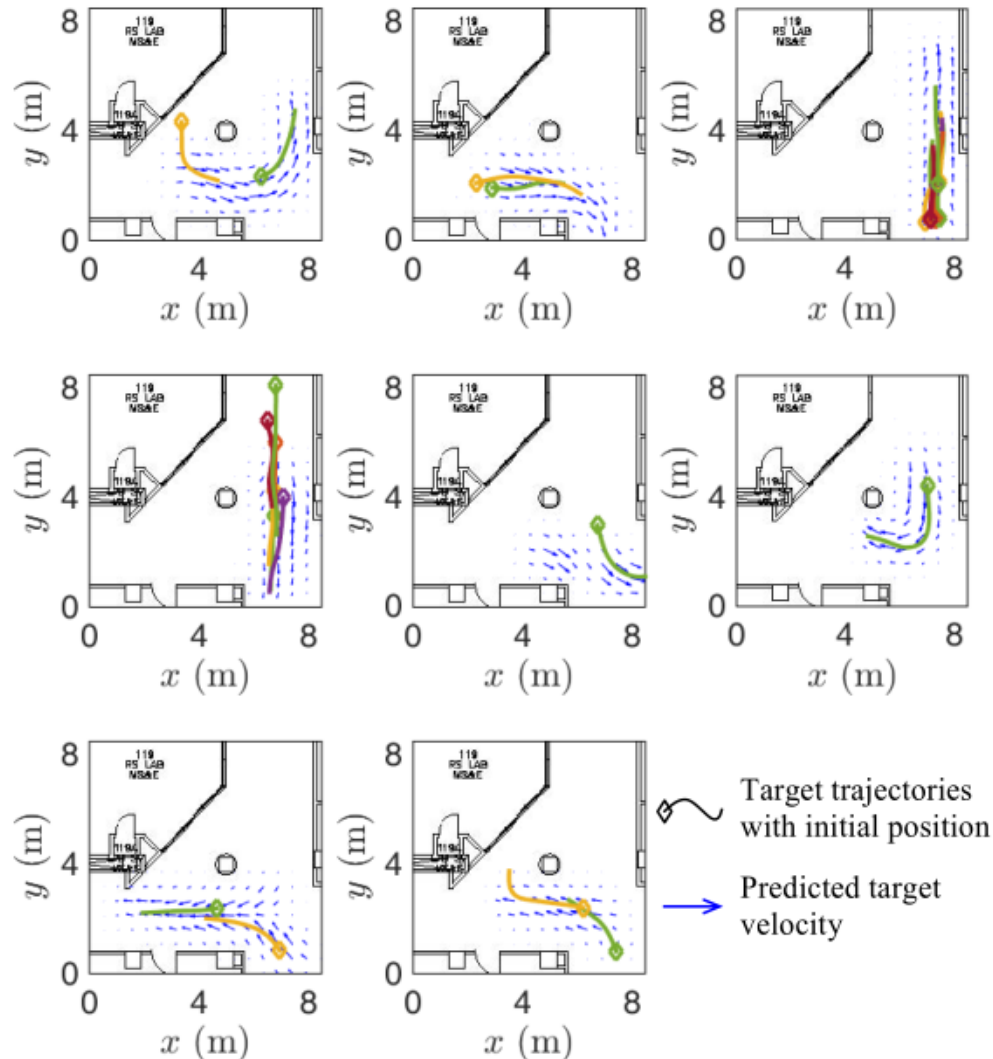
- Accumulated reward by different approaches



- cope with multiple targets
- limited sensor field of view
- aim at learning the change of target model

Simulation Result

- Final model obtained by the lexicographic algorithm compared with the testing target trajectories



Simulation Result

TABLE I
 ROOT MEAN SQUARE ERROR (RMSE) OF DPGP-MM

Algorithms	All data	Optimal solution	Lexicographic	Entropy reduction	Greedy	Potential field	Patrol	Random
Bldg4 data	8.97%	9.12%	9.15%	16.25%	15.68%	29.72%	27.47%	92.81%
Hallway data	9.03%	9.58%	10.88%	18.52%	17.89%	30.21%	40.17%	93.51%

$$\epsilon = \frac{1}{N} \sum_{j=1}^N \sum_{i=1}^M w_{ij} \sqrt{\frac{\delta t}{(t_{f_j} - t_{o_j})} \sum_{k=1}^{(t_{f_j} - t_{o_j})/\delta t} \frac{\|\mathbf{v}_j(k) - \hat{\mathbf{v}}_j(k)\|_2^2}{\|\mathbf{v}_j(k)\|}}$$

TABLE II
 COMPUTATIONAL COMPLEXITY

Algorithms	Theoretical complexity	Experimental complexity (s)	
		Bldg4 data	Hallway data
Optimal solution	NP	16.014	15.092
Lexicographic	$O([(L+k)^2 + T^2 d^3]MNT)$	0.081	0.073
Entropy reduction	$O((k^2 + T^2 d^3)MNT)$	0.077	0.072
Greedy	$O([(L+k)^2 + \log(MN)]MN)$	0.044	0.044
Information potential	$O([(L+k)^2 + d]MN)$	0.003	0.003
Patrol	$O(1)$	< 0.001	< 0.001
Random	$O((L+k)^2 MN + dT)$	0.002	0.002

Conclusion

- Information driven sensor planning
- Target dynamics modelling
 - GP regression for nonlinear dynamics
 - Dirichlet process for clustering
- Information value (DPGP-EKLD) as utility function
 - Free-flying sensor dynamics
 - Linear dynamics with constraints
- Future work
 - Nonlinear sensor dynamics model
 - Decentralized control

...



Thank you!