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Robust Flight Control via Minimum $\mathcal{H}_\infty$ Entropy Principle

Bo Fu * and Silvia Ferrari †

Robust control methods, such as $\mathcal{H}_\infty$ and μ-analysis, have been proven very effective at guaranteeing input-output stability in the face of random disturbances, parameter variations, and rapidly changing command inputs. However, in real-world applications, $\mathcal{H}_\infty$ objectives cannot capture all necessary control design specifications. $\mathcal{H}_2$-synthesis, for example, is better suited to represent regulation and pole placement objectives. Minimum entropy control is one approach recently proposed to represent mixed $\mathcal{H}_\infty/\mathcal{H}_2$ objectives. In this paper, a minimum entropy control design is developed for output-feedback longitudinal flight control, so as to meet both RMS gain and $\mathcal{H}_\infty$-performance specifications. The comparison with a mixed-objective control design obtained by Linear Matrix Inequalities (LMIs) demonstrates the effectiveness and robustness of the proposed minimum-entropy design.

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I. Nomenclature

q	=	pitch rate in radians per second
w	=	disturbance signal
z	=	output signal in radians
$\tilde{e}$	=	regulated feedback control input
$\tilde{u}$	=	regulated feedback control input
α	=	angle of attack in radians
δ	=	tailfin deflection command signal
γ	=	positive scalar, denote an upper bound
η	=	measured acceleration signal
η_c	=	reference acceleration signal
ω	=	frequency in rad
$\mathbf{u}$	=	$\mathcal{H}_\infty$ controller outputs
$\mathbf{w}$	=	$\mathcal{H}_\infty$ inputs
$\mathbf{x}$	=	$\mathcal{H}_\infty$ states
$\mathbf{y}$	=	$\mathcal{H}_\infty$ controller inputs
$\mathbf{z}$	=	$\mathcal{H}_\infty$ outputs
I_n	=	$n \times n$ identity matrix
$T_{\mathbf{w}\to\mathbf{z}}$	=	Transfer function matrix from $\mathbf{w}$ to $\mathbf{z}$
$E(T(s), \gamma)$	=	$\mathcal{H}_\infty$ entropy
Δ	=	scalar parameter uncertainty
Λ	=	Uniformly distributed random variable
$\|\| \cdot \|\|_2$	=	Matrix two norm
$\|\| \cdot \|\|_\infty$	=	Matrix infinity norm

II. Introduction

Classical robust control techniques such as $\mathcal{H}_\infty$ have been studied extensively in the literature [1–5], and have been successfully applied in many aeronautical and aerospace applications, including aircraft longitudinal flight control [6], satellite attitude control [7] [8], and atmospheric flight control of rockets [9]. $\mathcal{H}_\infty$ design seeks to satisfy desired bounds on the $\mathcal{H}_\infty$ norm of the input-output transfer function matrix, to guarantee robust input-output stability. Earlier

*Postdoctral Associate, Sibley School of Mechanical and Aerospace Engineering, 529 Upson Hall, Cornell University, Ithaca, New York 14853. Member, AIAA.

†Professor, Sibley School of Mechanical and Aerospace Engineering, 543 Upson Hall, Cornell University, Ithaca, New York 14853. Senior Member, AIAA

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studies focused on convex synthesis problems for which analytical solutions exist. Later work by Apkarian and Noll [10] introduced non-smooth optimization techniques which lead to solutions for non-convex synthesis problems, including large systems with up to hundreds of state variables. Using linear matrix inequalities (LMIs), it was also shown that adaptive neural network control systems can be synthesized to meet the same performance and stability guarantees as linear $\mathcal{H}_\infty$ control designs [1, 11, 12]. One of the advantages of $\mathcal{H}_\infty$ design over other robust control techniques is the ease of design and applicability to multi-input multi-output (MIMO) systems [4]. Minimum-entropy design is an approach recently proposed for the development of robust controllers that meet mixed $\mathcal{H}_\infty/\mathcal{H}_2$ objectives. The goal of this paper is to demonstrate a minimum-entropy controller for longitudinal flight control and compare it to a well known LMI design also developed for mixed multi-objective optimization.

III. Background on $\mathcal{H}_\infty$ minimum entropy controller design

Early developments in minimum-entropy design can be traced back to the work of Glover and Mustafa [13] [14] in which an ‘entropy-like’ function was defined for a system characterized by a transfer function matrix $T(s) \in \mathcal{RH}_\infty$, where $\mathcal{RH}_\infty$ denotes the real rational subspace of Hardy space $\mathcal{H}_\infty$ that consists of all strictly proper and real rational stable transfer function matrices, as follows,

$$E(T(s), \gamma, s_0) := -\frac{\gamma^2}{2\pi}\int_{\infty}^{\infty} \ln|\det(I - \gamma^{-2}T^*(j\omega)T(j\omega))|(\frac{s_0^2}{s_0^2+\omega^2})d\omega \tag{1}$$

where $\gamma, s_0 \in (0, \infty)$. The idea behind minimum-entropy design is to minimize the above entropy function while bounding the infinity norm of the system transfer function matrix, such that $||T(s)||_\infty < \gamma$, where γ is the finite gain or desired bound on $\mathcal{H}_\infty$ performance. What was later realized was that when $s_0 \to \infty$, this definition of entropy has a close relationship to the $\mathcal{H}_2$-norm and the $\mathcal{H}_\infty$-norm of $T(s)$. Furthermore, it is useful in practice because of the existence of closed-form control laws that meet corresponding objectives, for a given state-space formulation [13]. The quantity $E(T(s), \gamma, \infty)$ is referred to as γ-entropy, or $\mathcal{H}_\infty$ entropy in the literature. In this paper, we will adopt the latter and simply denote it as $E(T(s), \gamma)$.

There have been several attempts at interpreting the concept of $\mathcal{H}_\infty$ entropy. Boyd and Barratt [3] showed that for single input single output (SISO) systems with transfer function $T(s)$, the $\mathcal{H}_\infty$ entropy can be interpreted as the average power of the system output with random feedback when the system is driven by a white noise, namely,

$$E(T(s), \gamma) = \mathbb{E}_\Lambda||\frac{T(s)}{1-\Lambda T(s)}||_2^2 \tag{2}$$

where $\mathbb{E}_\Lambda$ denotes the expectation operator with respect to Λ, and Λ is a random variable uniformly distributed on the disk of radius $1/\gamma$ in the complex plane. For MIMO systems, such interpretation becomes ambiguous. Connections between $\mathcal{H}_\infty$ entropy and Shannon entropy showed that, for a multivariable discrete-time linear-time-invariant (LTI) system, the mutual information rate of the disturbance and output signal is equivalent to the $\mathcal{H}_\infty$ entropy of a transfer function matrix M that, however, is seemingly unrelated to the plant [15]. From the above discussion, it is clear that various aspects of $\mathcal{H}_\infty$ entropy control should be further investigated. As a first step, this paper follows the authors' previous work [1, 11, 12] in robust control and flight systems, and demonstrates the effectiveness of a robust controller design using $\mathcal{H}_\infty$ entropy based methods.

IV. Problem formulation

Consider a class of MIMO systems described by the block diagram shown in Figure 1, where the plant P and controller K are real rational transfer matrices. Both P and K are dynamic and linear, and afford state-space representations,

$$P : \begin{cases} \dot{\mathbf{x}} = A\mathbf{x}_K + B_1\mathbf{w} + B_2\mathbf{u} \\ \mathbf{z} = C_1\mathbf{x} + D_{11}\mathbf{w} + D_{12}\mathbf{u} \\ \mathbf{y} = C_2\mathbf{x} + D_{21}\mathbf{w} + D_{22}\mathbf{u} \end{cases} \tag{3}$$

$$K : \begin{cases} \dot{\mathbf{x}}_K = A_K\mathbf{x}_K + B_K\mathbf{y} \\ \mathbf{u} = C_K\mathbf{x}_K + D_K\mathbf{y} \end{cases} \tag{4}$$

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and are characterized by the transfer function matrices

$$P(s) : \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}$$

$$K(s) : \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix}$$

The control design considered in this paper seeks to find a linear dynamic controller K that meets the following specifications:

1) track a step command η_c with zero steady-state error;
2) track η_c with minimal overshoot and a desired time constant τ;
3) enforce adequate high-frequency roll off, stability, and L_2 performance for all normalized uncertainties $\Delta \in [-1, 1]$.

The problem solution obtained by minimum entropy principle is described in the next section and demonstrated numerically in Section VI by considering a well-known longitudinal flight control problem that requires both regulation and robustness.

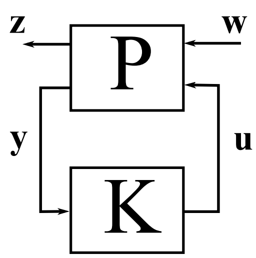

Fig. 1 Controller Block Diagram

V. Methodology

The minimum $\mathcal{H}_\infty$ entropy controller is also known as the central controller in the literature, and by definition is a sub-optimal $\mathcal{H}_\infty$ controller. As shown in [4], a minimum-entropy dynamic controller, K, can be designed to satisfy the following conditions:

1) Norm condition: $||T_{\mathbf{w}\to\mathbf{z}}||_\infty < \gamma$
2) Entropy condition: minimize the entropy function,

$$E(T_{\mathbf{w}\to\mathbf{z}}, \gamma) = -\frac{\gamma^2}{2\pi}\int_{-\infty}^{\infty} \ln|\det(I_n - \gamma^{-2}T^*_{\mathbf{w}\to\mathbf{z}}(j\omega)T_{\mathbf{w}\to\mathbf{z}}(j\omega))|d\omega \tag{5}$$

where $T^*_{\mathbf{w}\to\mathbf{z}}T_{\mathbf{w}\to\mathbf{z}}$ is an $n \times n$ transfer function matrix,.

It can be shown that $E(T_{\mathbf{w}\to\mathbf{z}}, \gamma) \geq 0$, and that the $\mathcal{H}_2$ objectives are implicitly stated by the entropy condition, as follows. First, consider the entropy function (1) in the limit of $\gamma \to \infty$, or,

$$\lim_{\gamma\to\infty} E(T_{\mathbf{w}\to\mathbf{z}}, \gamma) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_i \sigma_i^2(T_{\mathbf{w}\to\mathbf{z}}(j\omega))d\omega = ||T_{\mathbf{w}\to\mathbf{z}}||_2^2$$

where $\sigma_i(T_{\mathbf{w}\to\mathbf{z}})$ is the i'th singular value of $T_{\mathbf{w}\to\mathbf{z}}$. Next, let the transfer function matrix that satisfies both of the above criteria be denoted by $T^\dagger_{\mathbf{w}\to\mathbf{z}}$. Because $E(T^\dagger_{\mathbf{w}\to\mathbf{z}}, \gamma)$ is a monotonically decreasing function of γ (shown in [14]), we are sure to bound $||T^\dagger_{\mathbf{w}\to\mathbf{z}}||_2$ by some positive value γ_2,

$$||T^\dagger_{\mathbf{w}\to\mathbf{z}}||_2 = \sqrt{E(T^\dagger_{\mathbf{w}\to\mathbf{z}}, +\infty)} \leq \sqrt{E(T^\dagger_{\mathbf{w}\to\mathbf{z}}, \gamma)} := \gamma_2$$

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If it exists, a controller that meets the above criteria can be found by the approach outlined in [4]. It is assumed that the controller K stabilizes P internally. As a first step, the model of the plant is expressed in the standard $\mathcal{H}_\infty$ control structure as shown in Figure 1. Then, the plant is characterized by the state-space representation,

$$P : \begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + B_1\mathbf{w} + B_2\mathbf{u} \\ \mathbf{z} = C_1\mathbf{x} + D_{12}\mathbf{u} \\ \mathbf{y} = C_2\mathbf{x} + D_{21}\mathbf{w} \end{cases} \tag{6}$$

or equivalently in terms of transfer matrix $P(s)$,

$$P(s) : \begin{bmatrix} A & B_1 & B_2 \\ C_1 & Null & D_{12} \\ C_2 & D_{21} & Null \end{bmatrix}$$

where $Null$ is a zero matrix of the appropriate dimension. To design the controller, first construct two Hamiltonian matrices

$$H_\infty = \begin{bmatrix} A & \gamma^{-2}B_1B_1^* - B_2B_2^* \\ -C_1^*C_1 & -A^* \end{bmatrix}$$

and

$$J_\infty = \begin{bmatrix} A^* & \gamma^{-2}C_1^*C_1 - C_2^*C_2 \\ -B_1B_1^* & -A \end{bmatrix}$$

based on the above realization of the plant $P(s)$.

The relationship between a Hamiltonian matrix of the form

$$H := \begin{bmatrix} A_H & R_H \\ -Q_H & -A_H^* \end{bmatrix}$$

and its associated Riccati solution matrix X, is given by the algebraic Riccati equation:

$$A_H^*X + X_HA + XR_HX + Q_H = 0$$

As a result, it is possible to find solutions X_1 and X_2 to the corresponding algebraic Riccati equations associated with Hamiltonian matrices H_∞ and J_∞, respectively. Let $Ric : H \in \Re^{2n\times 2n} \mapsto X \in \Re^{n\times n}$ denote a mapping from an Hamiltonian matrix H to a solution X, such that the Riccati matrices X_1 and X_2 can be expressed in short-hand notation as,

$$X_1 = Ric(H_\infty)$$

and

$$X_2 = Ric(J_\infty)$$

As a second step, the Riccati matrices, X_1 and X_2, together with the transfer function matrix $P(s)$ are used to construct the following four matrices,

$$F_\infty := -B_2^*X_1 \tag{7}$$

$$L_\infty := -X_2C_2^* \tag{8}$$

$$Z_\infty := (I - \gamma^{-2}X_2X_1)^{-1} \tag{9}$$

$$\hat{A}_\infty := A + \gamma^{-2}B_1B_1^*X_1 + B_2F_\infty + Z_\infty L_\infty C_2 \tag{10}$$

and, subsequently, the minimum-entropy controller

$$K(s) : \begin{bmatrix} \hat{A}_\infty & -Z_\infty L_\infty \\ F_\infty & Null \end{bmatrix}$$

Finally, a controller, $K(s)$, that is in the above form and minimizes the entropy integral in 5 can be obtained by using an iterative process that also minimizes γ, as shown in [13].

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VI. Numerical Results

The $\mathcal{H}_\infty$ minimum-entropy control method described in Section V is demonstrated here for longitudinal flight control, and compared to a well-known robust controller obtained using mixed $\mathcal{H}_2/\mathcal{H}_\infty$ objectives by an LMI approach in [1]. The results show the effectiveness of the minimum-entropy design in meeting the performance goals defined in Section IV. They also show that the resulting minimum $\mathcal{H}_\infty$ entropy controller performance closely matches that of the classical LMI optimal controller design. Similar to what was carried out in reference [1], the software [16] was used to generate the reference LMI controller design. As the purpose of the LMI design is only to provide a reference and comparison to the $\mathcal{H}_\infty$ entropy design in this paper, the details of the LMI controller development will be omitted and only the performance of this controller will be stated.

Consider a highly maneuverable tailfin-controlled missile, described in [17] and [1]. Dynamic linear and nonlinar control designs have been obtained for this benchmark missile example using LMIs, μ-synthesis, and neural network synthesis, to name a few, and have been shown to successfully meet multiple control objectives, including $\mathcal{H}_2/\mathcal{H}_\infty$ performance in the presence of parameter uncertainty, and fast-changing command inputs. In this section, a robust controller with mixed objectives is obtained using the minimum $\mathcal{H}_\infty$ entropy approach, to meet a set of pre-defined mixed $\mathcal{H}_2/\mathcal{H}_\infty$ design objectives and command-input tracking objectives. The missile longitudinal dynamics can be described in terms of the longitudinal state vector, $\mathbf{x} = [\alpha \quad q]^T$, where α is the angle of attack and q is the pitch rate. The plant measurable output, $\mathbf{y} = [\eta \quad q]^T$, consists of the vertical acceleration, η, and the missile pitch rate. The missile longitudinal flight is controlled by virtue of the tail-fin deflection δ, and is subject to uncertain parameters comprised of aerodynamic coefficients, and represented by the random variable w. Assuming the angle of attack obeys $\alpha \in [0°,\ 20°]$, the linearized longitudinal dynamics of the tailfin-controller missile are given by

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -0.89 & 1 \\ -142.6 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 178.25 \end{bmatrix} w + \begin{bmatrix} -0.119 \\ -130.8 \end{bmatrix} \delta \tag{11}$$

$$\begin{bmatrix} \eta \\ q \end{bmatrix} = \begin{bmatrix} -1.52 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} - \begin{bmatrix} 0.203 \\ 0 \end{bmatrix} \delta \tag{12}$$

$$z = \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} \tag{13}$$

$$w = \Delta z \tag{14}$$

The output-feedback control structure proposed in [17] and [1], and illustrated by the block diagram in Fig. 2 is adopted, along with the shaping filters

$$W_e = 0.8$$

and

$$W_u = \frac{0.001s^3 + 0.03s^2 + 0.3s + 1}{0.00001s^3 + 0.03s^2 + 30s + 10000}$$

which can be used to incorporate bounds on the size of unmodeled dynamics and to penalize tracking error [17]. The normalized parameter uncertainty, $\Delta \in [-1, 1]$, represents variations in aerodynamic coefficients for the modeled operating range. Let the input vector be defined as $\mathbf{w} = [w, \eta_c]^T$, and the output be $\mathbf{z} = [z, \tilde{e}, \tilde{u}]^T$. Then, reformulate the plant dynamics using the $\mathcal{H}_\infty$ structure in Fig. 1. This results in a controller plant P that has six states: two from original plant G, three from the weighting filter W_u, and one from the integrator term ($\frac{1}{s}$). Finally, using the *hinfsyn* command from MATLAB® Robust control toolbox, a minimum $\mathcal{H}_\infty$ entropy controller is obtained and compared to the LMI control design in [1], as shown in Table 1.

	Minimum $\mathcal{H}_\infty$ entropy controller	**LMI controller**
$\mathcal{H}_\infty$ performance, γ	0.8047	0.8021
RMS gain	16.42	14.55

Table 1 Robust Performance Comparison

The open-loop controller response comparison is shown in Figure 3, where adequate high-frequency roll off is obtained. From the value of γ and the RMS gain, it is clear that the minimum $\mathcal{H}_\infty$ entropy design has comparable

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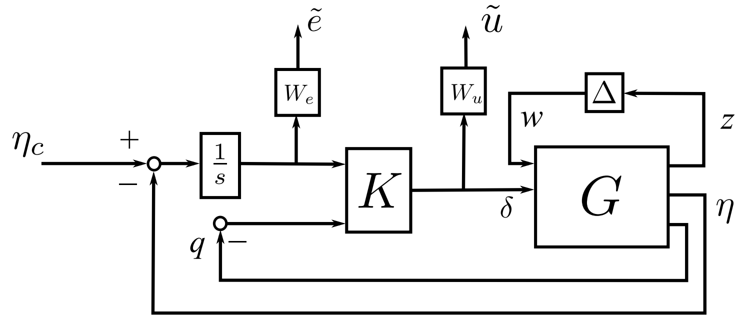

Fig. 2 Controller Feedback Block Diagram

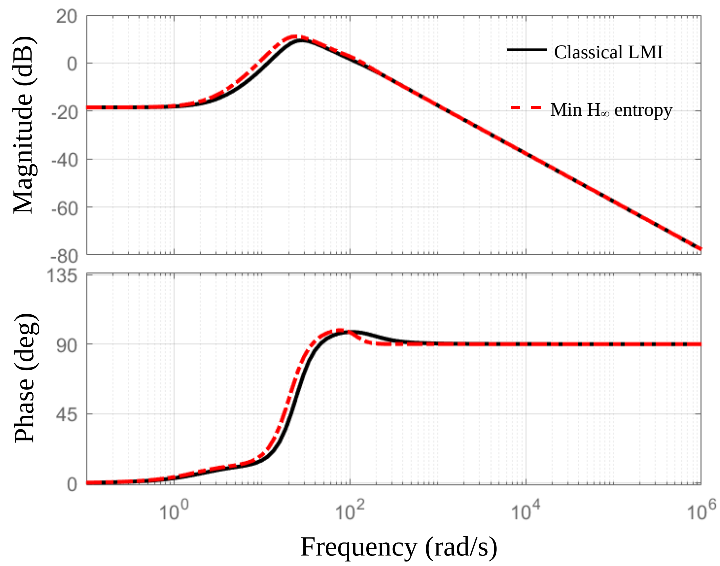

Fig. 3 Bode plot of open-loop response $G(s)K(s)$

robust performance to the LMI design. The closed-loop response of the controlled missile is shown in Figs. 4 and 5) for a step disturbance, demonstrating that the minimum $\mathcal{H}_\infty$ entropy controller meets the specification of tracking the command input in vertical acceleration (η_c) with minimal overshoot and a time constant of less than $\tau = 0.2$ (sec). The minimum $\mathcal{H}_\infty$ entropy design is a sub-optimal $\mathcal{H}_\infty$ design and offers, to date, no approach to improve transient response, as can be accomplished via pole placement by the LMI design. The system response comparison for a time-varying command input (η_c), plotted in Fig. 6, shows that the minimum $\mathcal{H}_\infty$ entropy controller can track a rapidly changing acceleration command with zero steady-state error. The controller states. $\dot{\mathbf{x}}_K$, and controller output, δ, are plotted in Fig. 7, where it can be seen that the output of the minimum $\mathcal{H}_\infty$ entropy controller is analogous to that of the LMI controller. Additionally, the closed-loop response of the missile controlled by $\mathcal{H}_\infty$ minimum-entropy principle is plotted in Figs. 8 and 9 using a Monte Carlo simulation over the range of the parameter uncertainty Δ for non-periodic and periodic command inputs, respectively. These results illustrate the closed-loop system robustness over the expected range of normalized uncertainty in aerodynamic coefficients, during regulatory control in time-varying command tracking problems.

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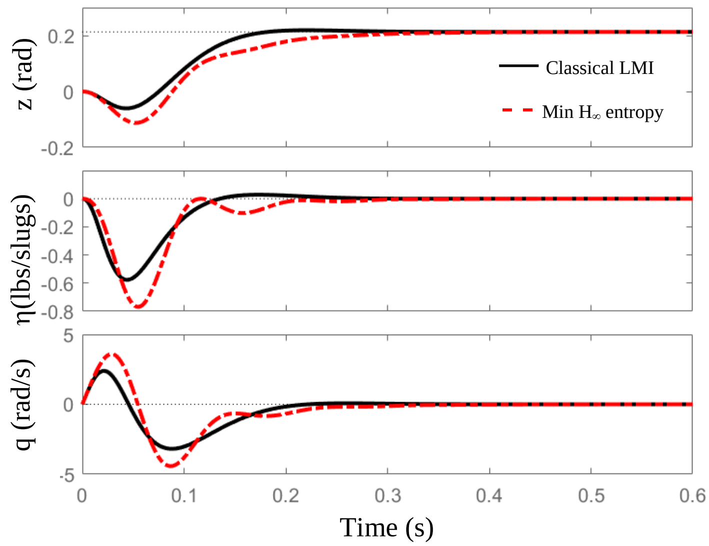

Fig. 4 Closed-loop system outputs for step disturbance

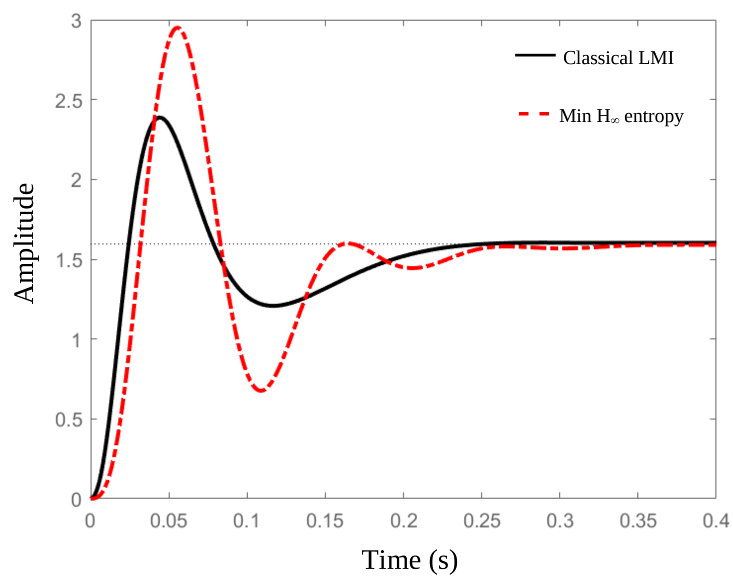

Fig. 5 Closed-loop control history *u* for step disturbance

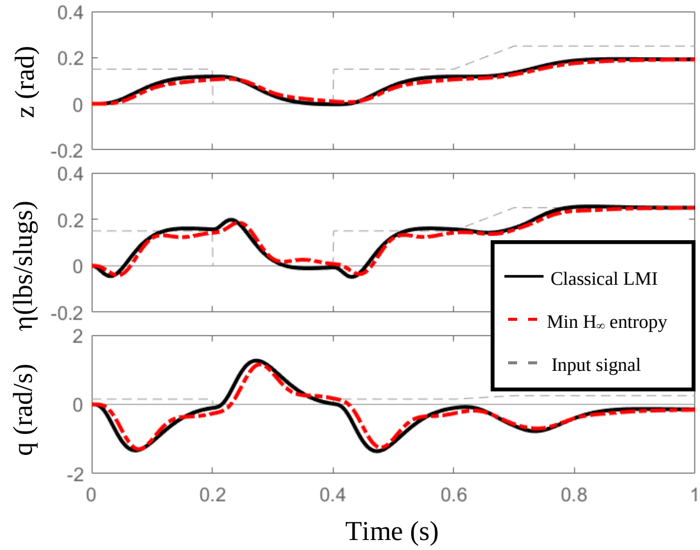

Fig. 6 Response of the linear system to command input η_c

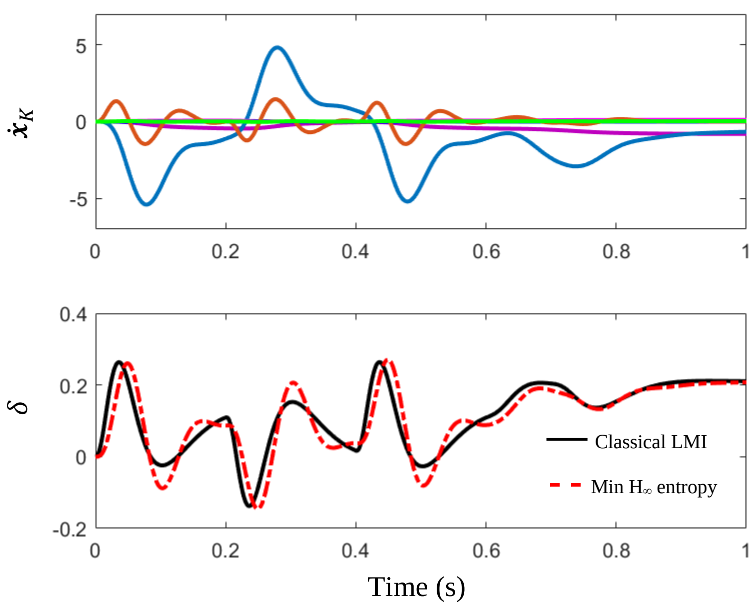

Fig. 7 Controller state derivatives $\dot{\mathbf{x}}_K$ and controller output δ

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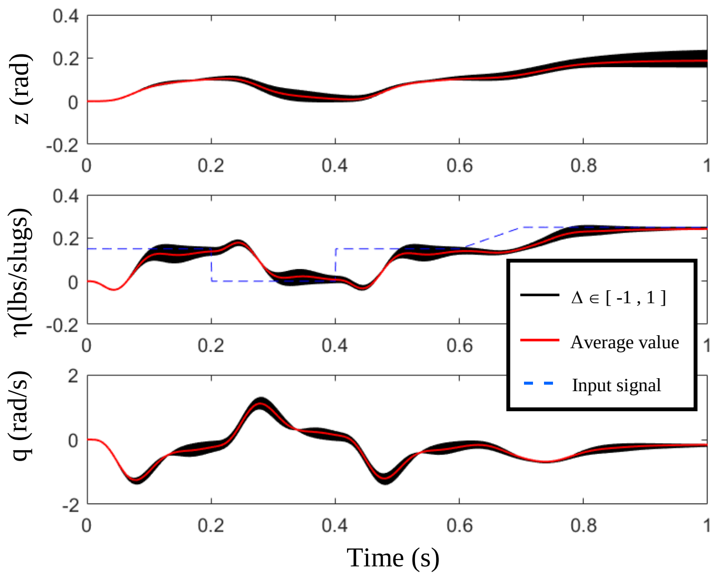

Fig. 8 $\mathcal{H}_\infty$ **entropy controller robustness for non-periodic command input** η_c

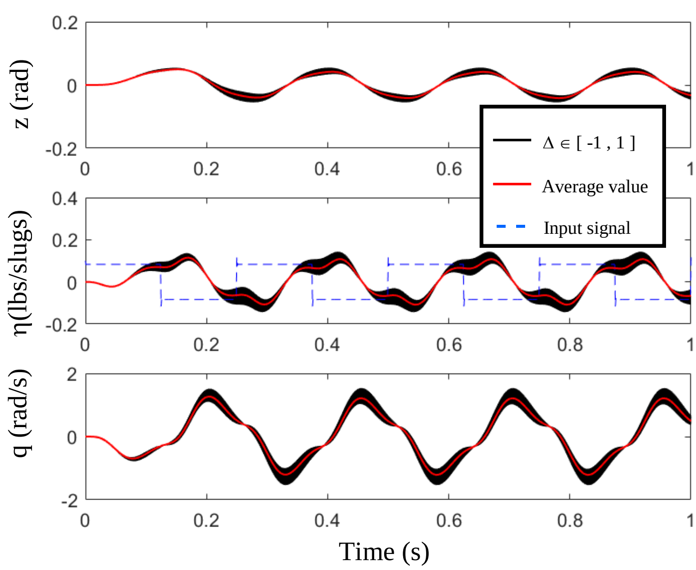

Fig. 9 $\mathcal{H}_\infty$ **entropy controller robustness for periodic command input** η_c

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VII. Conclusions and future work

This paper shows that a minimum $\mathcal{H}_\infty$ entropy controller can be developed to meet multiple control design specifications, as required by longitudinal flight control problems. The controller is demonstrated for robust control in the presence of time-varying command inputs and parameter variations. Furthermore, the minimum-entropy controller displays a performance similar to that of a robust controller developed for the same missile control problem using multi-objective $\mathcal{H}_\infty$-synthesis via LMIs. However, unlike the LMI solution, the minimum-entropy controller does not currently afford transient response specifications, such as pole placement. Future work will explore the use of other information theoretic functions to account for a broader range of control design specifications, and capture the Shannon information flow in the input and output channels. Subsequently, using neural network synthesis, these objective functions will be used to develop robust adaptive control structures with mixed objectives and performance guarantees.

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