A COMPARISON OF INFORMATION THEORETIC FUNCTIONS FOR TRACKING MANEUVERING TARGETS

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ABSTRACT

Several information theoretic functions have been proposed in the literature to assess the information value of sensor measurements *a posteriori*, that is, after measurements have been obtained from one or more targets. Sensor planning algorithms, however, require that the value of future sensor measurements be computed *a priori*, based on available models and prior information. An approach was recently presented by the authors for estimating the expected information value of future sensor measurements in target classification problems. The approach derives expected information theoretic functions from probabilistic models of the sensors and the targets, conditioned on prior information. In this paper, the approach is extended to the problem of sensor planning for tracking maneuvering targets. The approach is illustrated for a sensor that obeys an exponential power law model of received isotropic energy, and a target that obeys a Markov motion model. The performance of five information theoretic functions is compared through numerical simulations, and the results show that the objective function based on conditional mutual information leads to the most effective sensor planning strategy.

***Index Terms*—** Information theory, sensor, planning, target, tracking, mutual information.

1. INTRODUCTION

The problem of tracking maneuvering targets with little or no prior information is relevant to a variety of sensor applications, such as, monitoring of urban environments and facilities [1], and tracking anomalies in manufacturing plants. Sensor planning can be viewed as a decision making problem for an information-gathering agent that must decide a measurement sequence in order to optimize the sensing performance over time. Typically, the ability of a sensor to track a moving target with little or no prior information regarding target velocity and heading is not available in closed form. However, under proper assumptions, sensor tracking can be reduced to the problem of estimating the target state from partial or imperfect sensor measurements [2], by using a probability mass function (PMF) representation of the sensor measurements. Therefore, the sensor performance can be expressed as a function of the amount of information associated with the target state variables.

Although information theoretic functions can be used to quantify the amount of information associated with a random variable, they typically require knowledge of the random variable's probability mass (or density) function. Therefore, in order to utilize an information theoretic function to quantify the amount of information associated with a measurement sequence, the corresponding posterior distribution (also known as posterior belief) typically is computed from the measurement sequence. By this approach, Shannon entropy was used in [3] for tracking a moving target. Also, relative entropy was used in [2] to solve a multisensor-multitarget assignment problem, and in [4] to manage agile sensors with Gaussian models for target detection and classification.

The main difficulty in using existing information theoretic functions for sensor planning is that sensor decisions must be planned prior to obtaining the sensor measurements, while these information theoretic functions can only be evaluated after obtaining the sensor measurements [5]. A general approach was recently presented by the authors for estimating the *expected* information value of future sensor measurements in target classification problems [6]. The approach was also implemented to derive an additive, symmetric, and non-myopic function based on conditional mutual information for the detection and classification of landmines in [5], and for playing the game of CLUE® [7].

In this paper, the general approach presented in [6] is extended to the problem of tracking maneuvering targets, based on little or no prior information. The comparative performance of five information value functions derived from mutual information, Rènyi divergence, information potential, quadratic entropy, and the Cauchy-Schwartz [8] distance is analyzed numerically. The findings presented exhibit that the information value function based on conditional mutual

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information [7] results in the most effective sensor planning strategy for tracking moving targets.

2. PROBLEM FORMULATION

The target tracking problem considered in this paper involves a sensor deployed in a two-dimensional region of interest in order to track a set of N moving targets. The sensor measurements are modeled using an exponential power law that represents received isotropic energy generated by constant target source level, and attenuated by the environment. The targets are assumed to be point-masses modeled by piecewise Markov motion models, commonly used for multi-target tracking and estimation [9]. According to these Markov models, for every target i, with $i = 1, \ldots, N$, the motion can be assumed to be piece-wise linear and uniform during every time interval Δt_j, $j = 1, 2, \ldots$. Therefore, during Δt_j, the ith target track can be modeled as a straight line with an unknown velocity v_{ij}, an unknown heading angle θ_{ij}, and a known initial position $\mathbf{p}^0_{ij} = [x^0_{ij}, y^0_{ij}]^T$, such that,

$$\begin{aligned} x_{ij}(t) &= x^0_{ij} + v_{ij} t \cos\theta_{ij} \\ y_{ij}(t) &= y^0_{ij} + v_{ij} t \sin\theta_{ij}, \qquad t \in [0, \Delta t_j] \end{aligned} \tag{1}$$

for all $i = 1, \ldots, N$, and $j = 1, 2, \ldots$. An example of a target track modeled by (1) is shown in Fig. 1 for $j = 1, 2, 3$.

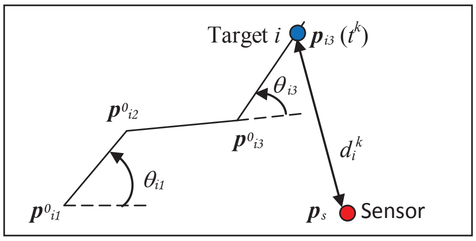

Fig. 1. Example of a target track modeled by (1).

It is assumed that during Δt_j the sensor can take only one measurement from one target, at a time $t^k \in \{t^1, \ldots, t^n\}$, with $\Delta t_{j-1} \leq t^1 < \ldots < t^n \leq \Delta t_j$. The sensor planning algorithm, therefore, must decide which target to measure at t^k, such that the accuracy of the estimates of $\mathbf{X}_{ij} = [v_{ij}\ \theta_{ij}]^T$ is maximized. During every Δt_j, v_{ij} and θ_{ij} are assumed to be independently and identically distributed (i.i.d.) random variables, with uniform probability distributions. Suppose the sample space of v_{ij} is $V = \{1, 2, 3, 4, 5\}$, and that of θ_{ij} is $\Theta = \{0, \frac{\pi}{6}, \frac{\pi}{3}\}$, and let $\mathcal{X}$ denote the product space $V \times \Theta$. Without loss of generality, the method is presented for $j = 1$, and x_{ij} will be denoted by x_i hereon for brevity.

The sensor position, $\mathbf{p}_s = [x_s\ y_s]^T$, is assumed to be known and constant, and a target is detected when the received signal exceeds a chosen threshold. The distance d^k_i between the ith target and the sensor can then be estimated from a measurement $\mathbf{Z}^k_i$ obtained according to the power law,

$$\mathbf{Z}^k_i = \beta ||d^k_i||^{-\rho} + \nu, \ i = 1, \ldots, N \tag{2}$$

and subject to additive, zero-mean Gaussian noise ν, which is fully specified by its standard deviation σ. In the above equation $\|\cdot\|$ denotes the L_2-norm, β is a known constant that depends on the target characteristics and is assumed same for all targets, and ρ is an attenuation coefficient that depends on the environmental conditions [10].

From (1), the distance between the sensor and the ith target at time t^k obeys,

$$d^k_i = \sqrt{(x^0_i + v_i t^k \cos\theta_i - x_s)^2 + (y^0_i + v_i t^k \sin\theta_i - y_s)^2} \tag{3}$$

Let $\lambda = [\beta, \rho, \sigma]^T$ denote a vector of known target and environmental parameters. From (2), the probability of obtaining a sensor measurement $\mathbf{z}$ at t^k is

$$p(\mathbf{Z}^k_i = \mathbf{z} | \mathbf{X}_i, \lambda) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\mathbf{z} - \beta(d^k_i)^{-\rho})^2}{2\sigma^2}} \tag{4}$$

where $p(\mathbf{Z}^k_i | \mathbf{X}_i, \lambda)$ is the abbreviation for the probability mass function (PMF) $p_{\mathbf{Z}^k_i}(\mathbf{z}^k_i | \mathbf{X}_i, \lambda)$. Therefore, the information functions discussed in the next section, can be used to evaluate the target information value, as shown in Section 4.

3. BACKGROUND

Information theoretic functions, reviewed comprehensively in [11], seek to measure the uncertainty of a discrete and random variable $\mathbf{X}$, with finite range $\mathcal{X}$, from its probability mass function (PMF) $p(\mathbf{x})$ for $\mathbf{x} \in \mathcal{X}$. The simplest information theoretic function, known as Shannon entropy, measures the uncertainty of $\mathbf{X}$, and is defined as

$$H(\mathbf{X}) = -\sum_{\mathbf{x} \in \mathcal{X}} p(\mathbf{x}) \log_2 p(\mathbf{x}). \tag{5}$$

Sensor planning strategies based on Shannon entropy are usually myopic, since they do not consider prior sensor measurements but only the present measurements in a sequence [12].

Another information function is Rényi information or α-divergence, which is defined

$$D_\alpha(p \parallel q) = \frac{1}{\alpha - 1} \ln \sum_{\mathbf{x} \in \mathcal{X}} p^\alpha(\mathbf{x})\, q^{1-\alpha}(\mathbf{x}) \tag{6}$$

where $q(\mathbf{x})$ is also known as current belief, and $p(\mathbf{x})$ as posterior belief. It was used for multi-sensors multi-target assignment in [2] to measure the difference between the two PMFs $q(\mathbf{x})$ and $p(\mathbf{x})$. A recent study showed that, for a sample target tracking problem, the Rènyi divergence is most effective when $\alpha = 0.5$ [4].

Mutual information function measures the information content of one random variable about another random variable [11], and can be made non-myopic by conditioning it on prior sensor measurements. The conditional mutual information of two random variables $\mathbf{X}$ and $\mathbf{Z}$, given $\mathbf{Y}$, is given by

$$I(\mathbf{X};\mathbf{Z} \mid \mathbf{Y}) = H(\mathbf{X} \mid \mathbf{Y}) - H(\mathbf{X} \mid \mathbf{Z},\mathbf{Y}) = \sum_{\mathbf{x}\in\mathcal{X}}\sum_{\mathbf{y}\in\mathcal{Y}}\sum_{\mathbf{z}\in\mathcal{Z}} p(\mathbf{x},\mathbf{y},\mathbf{z}) \log_2 \frac{p(\mathbf{x},\mathbf{z} \mid \mathbf{y})}{p(\mathbf{x} \mid \mathbf{y})p(\mathbf{z} \mid \mathbf{y})} \quad (7)$$

represents the reduction in uncertainty in $\mathbf{X}$ due to knowledge of $\mathbf{Z}$, when $\mathbf{Y}$ is given.

The information function based on the Cauchy-Schwartz (CS) inequality is defined by [8],

$$C(p,q) = \log_2 \frac{\sum_{\mathbf{x}\in\mathcal{X}} p^2(\mathbf{x}) \sum_{\mathbf{x}\in\mathcal{X}} q^2(\mathbf{x})}{\left[\sum_{\mathbf{x}\in\mathcal{X}} p(\mathbf{x})q(\mathbf{x})\right]^2} \quad (8)$$

which measures the difference between two PMFs $p(\mathbf{x})$ and $q(\mathbf{x})$, was recently implemented for sensor planning in [6].

As shown in [6], when compared to all other information theoretic functions, conditional mutual information (7) and quadratic entropy, defined as,

$$V(\mathbf{X}) = \sum_{\mathbf{x}\in\mathcal{X}} p^2(\mathbf{x}) \quad (9)$$

typically lead to the most effective objective functions for sensor planning in target classification problems. In the next section, the above information theoretic functions are used to derive information value functions for the target tracking problem presented in Section 2, and their performance is compared in Section 5.

4. INFORMATION VALUE FUNCTIONS FOR SENSOR PLANNING

This section presents a methodology for computing the information value of a target prior to obtaining sensor measurements. By this approach, at every time step t^k, the sensor can decide to obtain measurements from the target with the highest information value, in an effort to optimize its tracking performance. As shown in Section 3, computing the uncertainty of a random variable, or the difference in uncertainty before and after obtaining sensor measurements, requires knowledge of the prior and posterior belief states. Since the posterior belief is unknown prior to obtaining the measurements, the approach presented in [6] is extended here to compute the *expected* information value of future sensor measurements, in the presence of multiple moving targets.

Consider a sensor that must decide which target to measure at time t^k, based on the set of all prior measurements $\mathcal{M}_i^{k-1} = \{\mathbf{Z}_i^1, \ \ldots, \ \mathbf{Z}_i^{k-1}\}, \ i = 1,\ldots,N$, where any entry can be $\emptyset$ if the corresponding measurement is missing. The information value of target i can be represented by the change in belief state brought about by $\mathbf{Z}_i^k$, as measured by the α-divergence in (6). At time t^k, the change between the prior belief state, $p(\mathbf{X}_i \mid \mathcal{M}_i^{k-1},\lambda)$, and the posterior belief state, $p(\mathbf{X}_i \mid \mathbf{Z}_i^k, \mathcal{M}_i^{k-1},\lambda)$, can be estimated by taking the expectation with respect to $\mathbf{Z}_i^k$, denoted by $\mathbb{E}_{\mathbf{Z}_i^k}$. Then, from (6), the information value can be represented by the *expected α-divergence*,

$$\begin{aligned}
&\hat{\varphi}_{D_\alpha}(\mathbf{X}_i;\mathbf{Z}_i^k \mid \mathcal{M}_i^{k-1},\lambda) \\
&\equiv \mathbb{E}_{\mathbf{Z}_i^k}\left\{D_\alpha[p(\mathbf{X}_i \mid \mathbf{Z}_i^k,\mathcal{M}_i^{k-1},\lambda) \parallel p(\mathbf{X}_i \mid \mathcal{M}_i^{k-1},\lambda)]\right\} \\
&= \sum_{\mathbf{z}\in\mathcal{Z}} D_\alpha[p(\mathbf{X}_i \mid \mathbf{Z}_i^k = \mathbf{z},\mathcal{M}_i^{k-1},\lambda) \parallel p(\mathbf{X}_i \mid \mathcal{M}_i^{k-1},\lambda)] \\
&\times p(\mathbf{Z}_i^k = \mathbf{z} \mid \mathcal{M}_i^{k-1},\lambda) \qquad (10)
\end{aligned}$$

By taking the expectation with respect to $\mathbf{Z}_i^k$, the measurement value $\mathbf{z}$ is no longer needed, and $\hat{\varphi}_{D_\alpha}$ can be computed from $\mathcal{M}_i^{k-1}$ and the sensor model, as explained below.

As shown in [4], measurements that are obtained at different times can be assumed to be conditionally independent given the state, i.e. $p(\mathbf{Z}_i^{k-1} \mid \mathbf{X}_i, \mathbf{Z}_i^{k-2},\ldots,\mathbf{Z}_i^1,\lambda) = p(\mathbf{Z}_i^{k-1} \mid \mathbf{X}_i,\lambda)$. Thus, the measurement distribution at t^k is

$$p(\mathbf{Z}_i^k = \mathbf{z}|\mathcal{M}_i^{k-1},\lambda) = \sum_{\mathbf{X}_i\in\mathcal{X}} p(\mathbf{Z}_i^k = \mathbf{z}|\mathbf{X}_i)p(\mathbf{X}_i|\mathcal{M}_i^{k-1},\lambda). \quad (11)$$

The posterior belief inside the expectation in (10) can be calculated using Bayes' rule for every value $\mathbf{z} \in \mathcal{Z}$,

$$p(\mathbf{X}_i \mid \mathbf{Z}_i^k = \mathbf{z},\mathcal{M}_i^{k-1},\lambda) = \frac{p(\mathbf{z} \mid \mathbf{X}_i,\lambda)p(\mathbf{X}_i \mid \mathcal{M}_i^{k-1},\lambda)}{\sum_{\mathbf{x}_\ell\in\mathcal{X}} p(\mathbf{z} \mid \mathbf{X}_i = \mathbf{x}_\ell,\lambda)p(\mathbf{X}_i = \mathbf{x}_\ell \mid \mathcal{M}_i^{k-1},\lambda)} \quad (12)$$

such that, expected α-divergence (10) can be computed from (11) and (12). Similarly, based on the definition of conditional mutual information in (7), the *expected conditional mutual information* can be obtained as follows,

$$\begin{aligned}
&\hat{\varphi}_I(\mathbf{X}_i;\mathbf{Z}_i^k \mid \mathcal{M}_i^{k-1},\lambda) \equiv \mathbb{E}_{\mathbf{Z}_i^k}\left\{I(\mathbf{X}_i;\mathbf{Z}_i^k \mid \mathcal{M}_i^{k-1},\lambda)\right\} \\
&= H(\mathbf{X}_i \mid \mathcal{M}_i^{k-1},\lambda) - \sum_{\mathbf{z}\in\mathcal{Z}} H(\mathbf{X}_i \mid \mathbf{Z}_i^k = \mathbf{z},\mathcal{M}_i^{k-1},\lambda) \\
&\times p(\mathbf{Z}_i^k = \mathbf{z} \mid \mathcal{M}_i^{k-1},\lambda). \qquad (13)
\end{aligned}$$

where, the entropy $H(\mathbf{X}_i \mid \mathbf{Z}_i^k = \mathbf{z},\mathcal{M}_i^{k-1},\lambda)$ is computed from (12), using (5).

The expected Cauchy-Schwartz information function, derived from (8), is,

$$\begin{aligned}
&\hat{\varphi}_C(\mathbf{X}_i;\mathbf{Z}_i^k \mid \mathcal{M}_i^{k-1},\lambda) \\
&\equiv \mathbb{E}_{\mathbf{Z}_i^k}\left\{C[p(\mathbf{X}_i \mid \mathbf{Z}_i^k,\mathcal{M}_i^{k-1},\lambda), p(\mathbf{X}_i \mid \mathcal{M}_i^{k-1},\lambda)]\right\} \\
&= \sum_{\mathbf{z}\in\mathcal{Z}} C[p(\mathbf{X}_i \mid \mathbf{Z}_i^k = \mathbf{z},\mathcal{M}_i^{k-1},\lambda), p(\mathbf{X}_i \mid \mathcal{M}_i^{k-1},\lambda)] \\
&\times p(\mathbf{Z}_i^k = \mathbf{z} \mid \mathcal{M}_i^{k-1},\lambda) \qquad (14)
\end{aligned}$$

and is an alternative measure of the distance between the prior and the posterior belief state for a target i, prior to obtaining the measurement value $\{\mathbf{Z}_i^k = \mathbf{z}\}$. The information value of the ith target can also be represented by the *expected information potential gain*,

$$\begin{aligned}
&\hat{\varphi}_V(\mathbf{X}_i; \mathbf{Z}_i^k \mid \mathcal{M}_i^{k-1}, \lambda) \\
&\equiv \mathbb{E}_{\mathbf{Z}_i^k}\left\{V[p(\mathbf{X}_i \mid \mathbf{Z}_i^k, \mathcal{M}_i^{k-1}, \lambda)] - V[p(\mathbf{X}_i \mid \mathcal{M}_i^{k-1}, \lambda)]\right\} \\
&= \sum_{\mathbf{z} \in \mathcal{Z}} V[p(\mathbf{X}_i \mid \mathbf{Z}_i^k = \mathbf{z}, \mathcal{M}_i^{k-1}, \lambda)] p(\mathbf{Z}_i^k = \mathbf{z} \mid \mathcal{M}_i^{k-1}, \lambda) \\
&- V[p(\mathbf{X}_i \mid \mathcal{M}_i^{k-1}, \lambda)] \qquad (15)
\end{aligned}$$

derived from the information potential in (9). The *expected quadratic entropy reduction* is similarly obtained,

$$\begin{aligned}
&\hat{\varphi}_{H_{R_2}}(\mathbf{X}_i; \mathbf{Z}_i^k \mid \mathcal{M}_i^{k-1}, \lambda) \\
&\equiv \mathbb{E}_{\mathbf{Z}_i^k}\left\{H_{R_2}[p(\mathbf{X}_i \mid \mathcal{M}_i^{k-1}, \lambda)] - H_{R_2}[p(\mathbf{X}_i \mid \mathbf{Z}_i^k, \mathcal{M}_i^{k-1}, \lambda)]\right\} \\
&= H_{R_2}[p(\mathbf{X}_i \mid \mathcal{M}_i^{k-1})] - \sum_{\mathbf{z} \in \mathcal{Z}} H_{R_2}[p(\mathbf{X}_i \mid \mathbf{Z}_i^k = \mathbf{z}, \mathcal{M}_i^{k-1}, \lambda)] \\
&\times p(\mathbf{Z}_i^k = \mathbf{z} \mid \mathcal{M}_i^{k-1}, \lambda) \qquad (16)
\end{aligned}$$

where $H_{R_2}(X) = -\ln V(x)$. Both (15) and (16) can be computed from (12) and (5), using the sensor model (4) and the prior measurements $\mathcal{M}_i^{k-1}$.

5. NUMERICAL SIMULATIONS AND RESULTS

In this section, the information value functions are utilized to select the measurement sequence in the target tracking problem described in Section 2. The noise in (4) is modeled using a standard deviation $\sigma = 0.3$. Five greedy sensor planning strategies are implemented by maximizing each of the information value functions presented in Section 4. After the measurements are obtained, the actual sensor performance is evaluated using the tracking error,

$$e = \sum_{i=1}^{n} \sqrt{(\hat{p}_i^c - p_i)^2 + (\hat{q}_i - q_i)^2} \qquad (17)$$

where $[p_i\ q_i]^T \equiv [v_i\ \theta_i]^T$ are the actual target parameters, and $[\hat{p}_i\ \hat{q}_i]^T \equiv [\hat{v}_i\ \hat{\theta}_i]^T = \mathbf{X}_i$ are the estimated values.

The tracking errors are averaged over 40 time intervals Δt_j. The results, summarized in Fig. 2, show that with all of the value functions presented in Section 4 the tracking error e decreases over time, thereby producing an effective measurement sequence. In particular, the expected conditional mutual information, defined in (13), outperforms other functions in that it leads to the fastest and, overall, greatest decrease in e.

6. CONCLUSIONS

An approach is presented for estimating the information value of future sensor measurements in sensor planning for tracking maneuvering targets. The approach derives expected information value functions from probabilistic models of the sensors and the targets, conditioned on prior information. Five value functions are derived and implemented to select the measurement sequence with the best tracking performance. It was found that all value functions lead to a reduction of the tracking error over time, and that expected conditional mutual information constitutes the most effective value function.

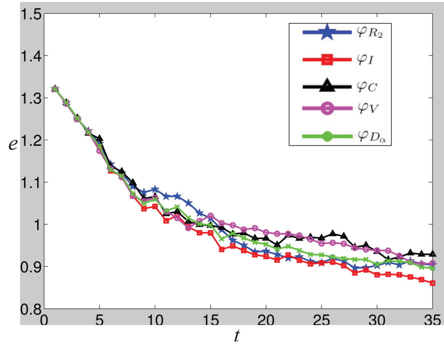

Fig. 2. Time history of average tracking errors

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