ONR Maritime Sensing - Discovery & Invention (D&I) Review
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Multi-scale Adaptive Sensor Systems

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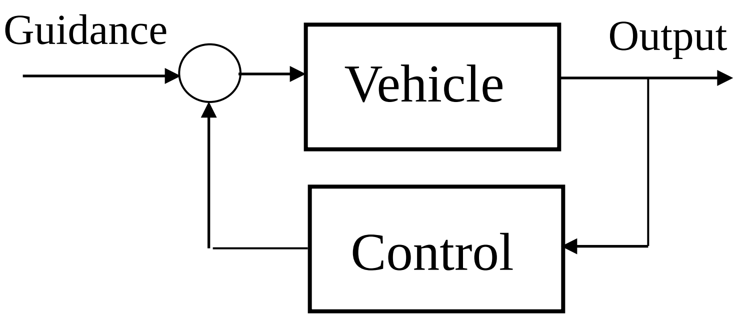
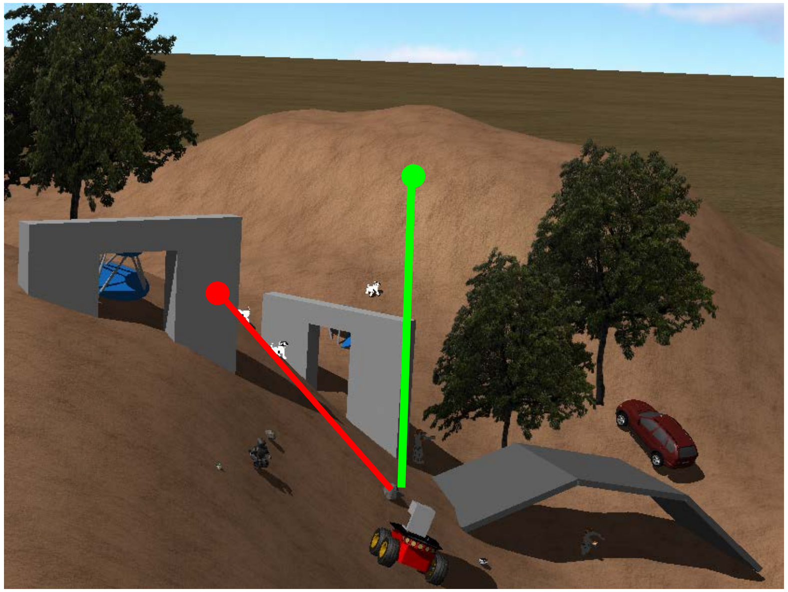
Vehicle Navigation and Control

Traditional paradigm:

Sensor information (output) used as feedback to the vehicle to support the **vehicle navigation objectives**.



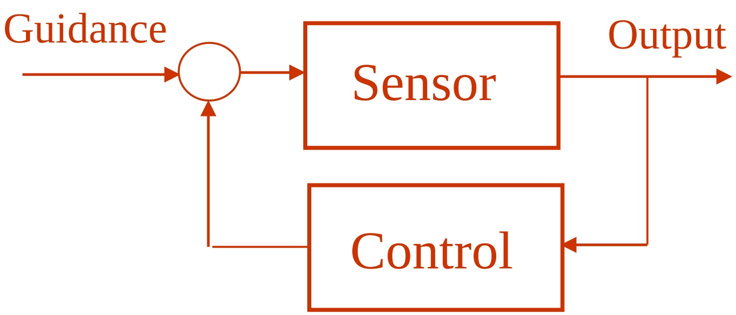


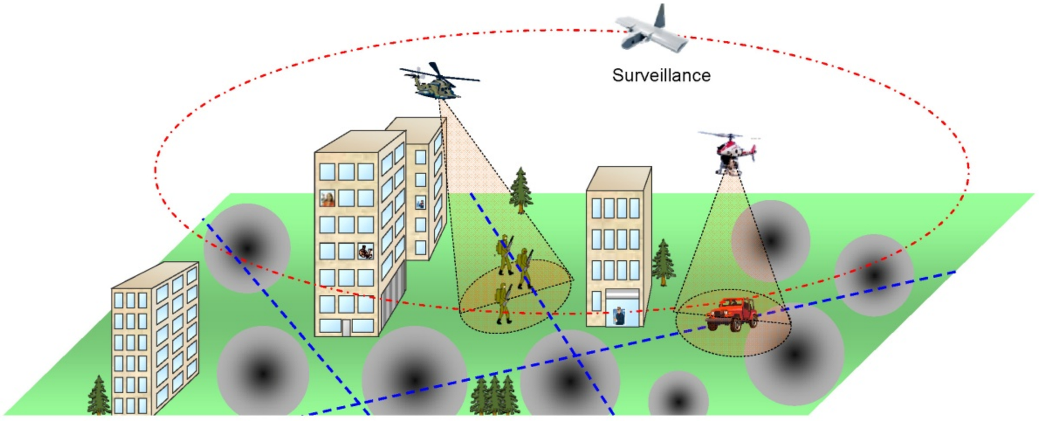
Prior research foci:

- **Sensor-based path planning**
  Navigation sensors for obstacle avoidance
  Simultaneous localization and mapping (SLAM)
- **Dual/exploratory control**
  System identification
- **Output-based feedback control**

Sensor Navigation and Control

New paradigm:
Vehicle is used to gather information (output) to support **sensing objectives**, such as coverage or target DCLT.



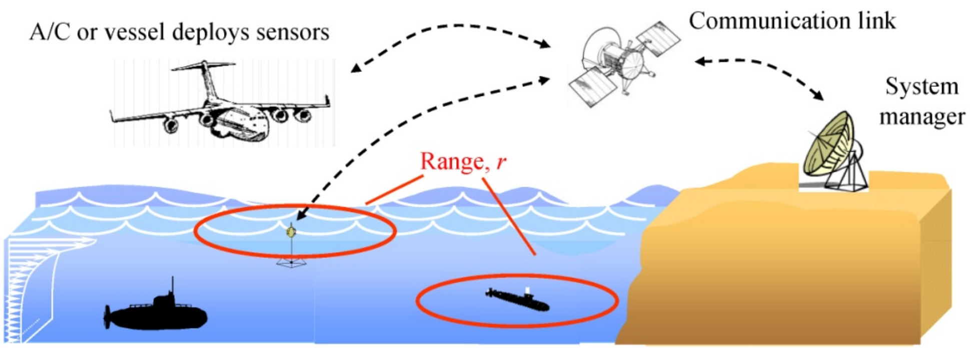


Information-driven sensor navigation and control
Trajectory planning and feedback control based on information value, target and sensor geometries, and platform kinematic/dynamic constraints.

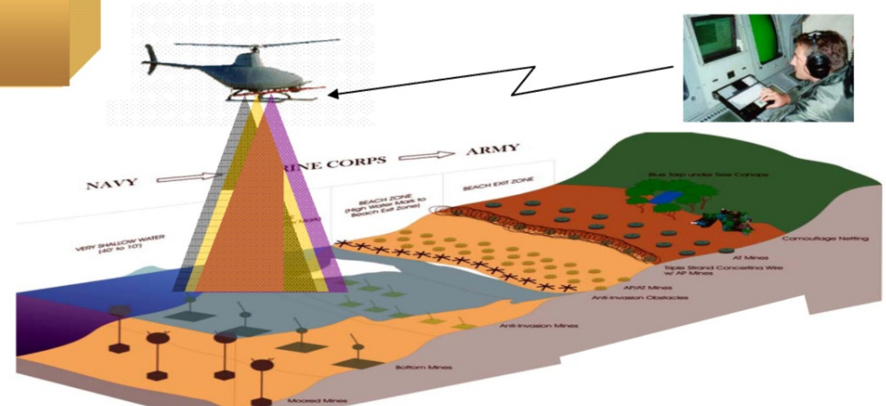
S. Ferrari and T. A. Wettergren, *Information-driven planning and Control*, CRC Press, Boca Raton, FL, scheduled to appear 2018.

Modern Sensor Systems

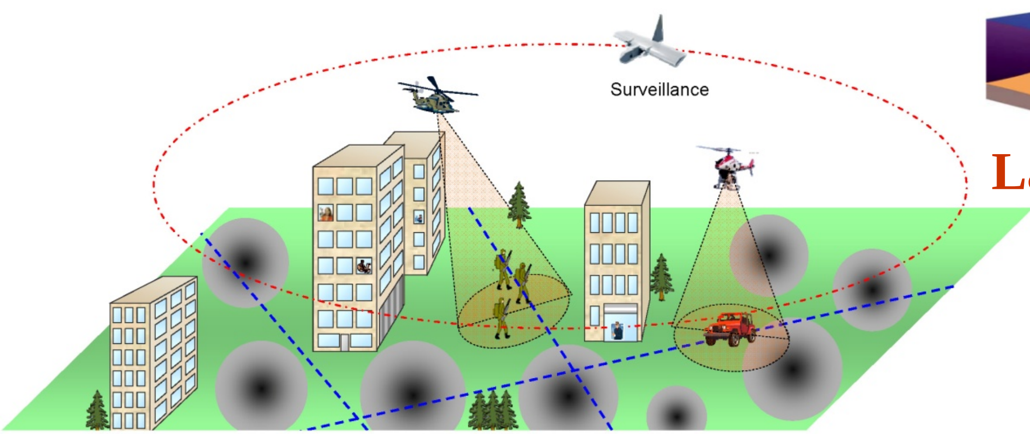
Multiple sensors installed on mobile vehicles for information gathering.



Undersea surveillance



Landmine detection and identification



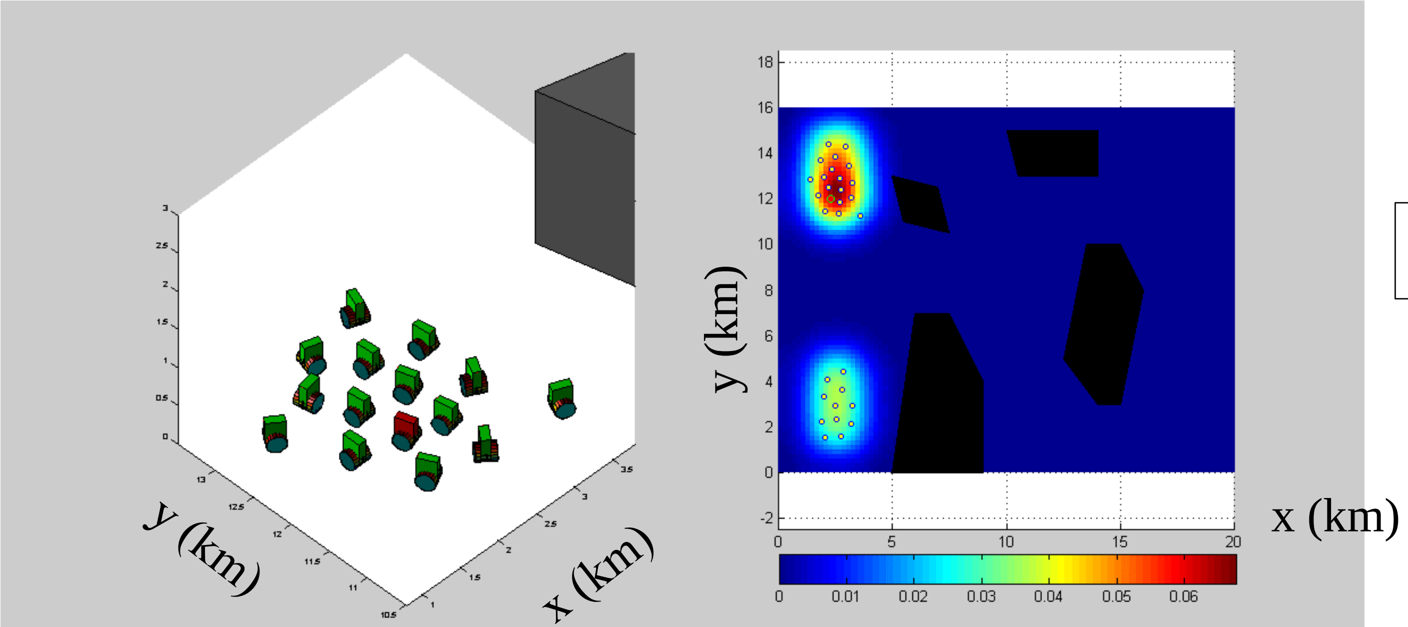
Monitoring of urban environments

Distributed Optimal Control (DOC)

- Agents operating in Region of Interest (ROI) $W \subset \mathbb{R}^2$
- Performance measured in terms of restriction operator $\wp(\mathbf{x}_i,t)$
- Restriction Operator $\wp: W \times \mathbb{R} \rightarrow \mathbb{R}$
  - Time varying PDF $\wp(\mathbf{x}_i,t)$
  - PDF-based control law $\mathbf{u}_i\,(t)$

Terminal Cost — Instantaneous cost (Lagrangian)

$$J = \phi[\wp(\mathbf{x}_i, T_f)] + \int_{T_0}^{T_f}\int_X \mathscr{L}[\wp(\mathbf{x}_i, t), \mathbf{u}_i(t), t]\, d\mathbf{x}_i\, dt$$



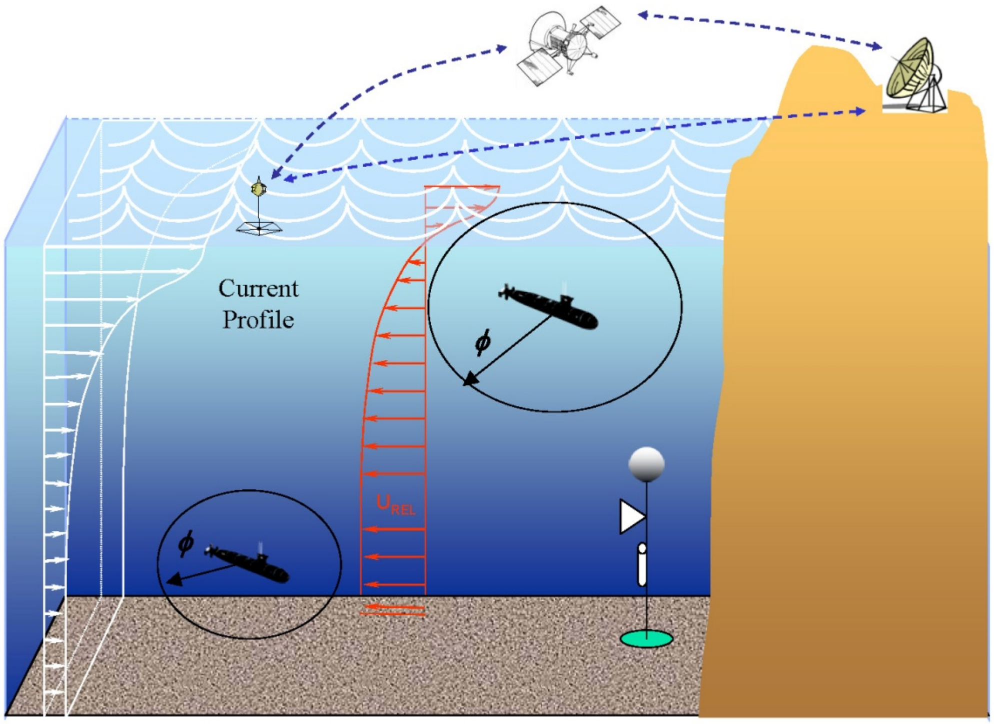
Agent Density, PDF $\wp(\mathbf{x}_i,t)$

Environment:

■ Obstacle

Technical Challenges (ASW)

- Environmental and operating conditions *in situ* may drastically differ from those used *a priori*
  - – Off-line DOC solutions no longer optimal
  - – Agents must react to local information, including new tactical constraints
  - – Limited communications (covertness) and inertial positioning systems
  - – Network-level controller can dispatch agents to localize gradient intensification while providing energy management, volume coverage, and robustness to component failure



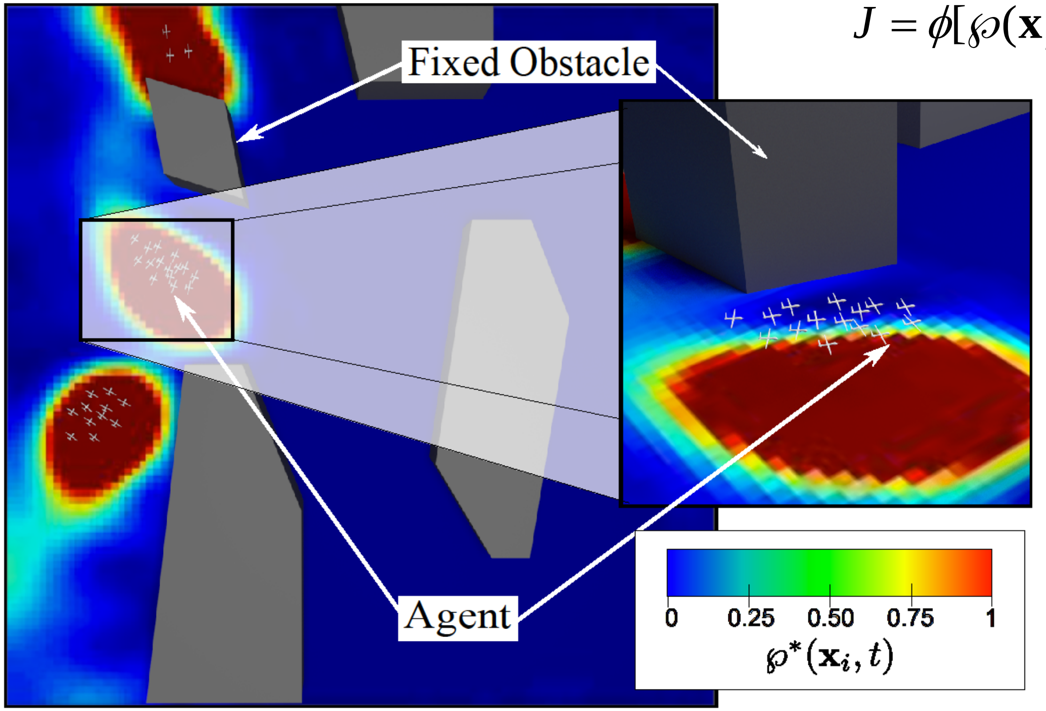
1. **Target:** actual population is different from that assumed *a priori*.
2. **Environment:** conditions measured in situ are different from those forecasted by oceanographic models.
3. **Platform:** navigation settings are suboptimal, leading to incorrect estimates of agent position and/or direction.
4. **Sensor:** actual performance is different from the performance function model due to the above conditions, or sensor malfunctioning.

Technical Developments (Yr 2)

- Optimal and minimum number of agents to meet mission requirements (FISST)
- Information value and discrimination with limited communications
- Multi-scale environmental adaptation

Optimal Time-varying Probability Density Function (PDF), $\wp^*(\mathbf{x}_i)$: max J

$$J = \phi[\wp(\mathbf{x}_i, T_f)] + \int_{T_0}^{T_f} \int_X \mathscr{L}[\wp(\mathbf{x}_i, t), \mathbf{u}_i(t), t] d\mathbf{x}_i dt$$



Macroscopic state (PDF):

$$\wp(\mathbf{x}_i, t)$$

Microscopic state of ith agent:

$$\mathbf{x}_i = [x, y, \theta]$$

Agent dynamics:

$$\dot{\mathbf{x}}_i = f[\mathbf{x}_i, \mathbf{u}_i, t]$$

DOC Analysis via Finite Set Statistics (FISST)

DOC and FISST Relationship

Use Finite Set Statistics (FISST) to rigorously analyze DOC results:

- In DOC, the agent microscopic states are viewed as random variables, with a possibly infinite number of agents.
- FISST applies to a finite number of agents in the ROI (in the limit of $N = 1$) that is possibly unknown and changing over time (births, deaths, ..).
- If the number of the agents is given, a multi-object probability density (MPDF) and the probability hypothesis density (PHD) of the agents in the DOC problem only depend on the spatial PDF of agents, or DOC macroscopic state.
- The propagation and update of the MPDF of the agents can be implemented by the FISST methods, like the PHD and the cardinalized PHD (or CPHD) filters.
- FISST, originally developed for multi-target multi-sensor tracking problems, extends probability theory and probability calculus to finite random sets (FRS).

Background on Finite Set Statistics

- Random Finite Sets (RFS)

Let $\mathcal{X}$ be an underlying space, such as a state space or a measurement space.

Then $\mathcal{X}^{\infty}$ denotes the power set of $\mathcal{X}$, which is the set of all subsets of $\mathcal{X}$.

A *random finite set* (RFS) is a random variable Ψ on $\mathcal{X}^{\infty}$.

- Power Set

The power set of a set $\mathcal{X} = \{x_1, x_2, x_3\}$ is the set of all finite subsets of $\mathcal{X}$:

$$\mathcal{X}^{\infty} = \{\emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}\}$$

Therefore, a realization of RFS Ψ can be:

$$X = \{\emptyset\}$$

$$X = \{x_1\}$$

$$X = \{x_1, x_2\}$$

$$\vdots$$

Multi-object Probability and Calculus

- Set Integral

Let $f(X)$ be a set function defined on $\mathcal{X}$. Then its *set integral* is defined as

$$\int_{\mathcal{T}} f(X)\delta X \equiv f(\emptyset) + \sum_{n=1}^{\infty} \frac{1}{n!} \int_{\mathcal{T}\times\ldots\times\mathcal{T}} f(\{\mathbf{x}_1,\ldots,\mathbf{x}_n\}) d\mathbf{x}_1 \ldots \mathbf{x}_n$$

where $\mathcal{T} \subset \mathcal{X}$

- Multi-object Probability Density Function (MPDF)

The set function defined $\mathcal{X}$ on is a multi-object probability density function if

- It is non-negative

$$f(X) \geq 0$$

- Its set integral is equal to 1

$$\int_{\mathcal{X}} f(X)\delta X = 1$$

Finite Set Statistics (FISST)

- Probability Hypothesis Density (PHD)

The probability hypothesis density of an RFS Ψ is a density function $D_\Psi(\mathbf{x})$ on single objects $\mathbf{x} \in \mathcal{X}$ which is defined by:

$$D_\Psi(\mathbf{x}) = \int_X f_\Psi(\{\mathbf{x} \cup W)\delta W = \frac{\delta G_\Psi}{\delta \mathbf{x}}[\mathbf{1}] = \frac{\delta \beta_\Psi}{\delta \mathbf{x}}(X)$$

where the number $D_\Psi(\mathbf{x})$ is the density of objects at $\mathbf{x}$.

- Cardinality Distribution

The cardinality distribution of an RFS $\Psi \subset \mathcal{X}$ is

$$p_\Psi(n) = P(|\Psi| = n) = \int_{|X|=n} f_\Psi(X)\delta X$$

Example: Multi-Bernoulli RFS

• Consider a simple example, where the RFS is represented by two random objects with respective PDFs $f_i(\mathbf{x})$ $(i=1,2)$ and the corresponding probability of existence q_i $(i=1,2)$. Then, the MPDF can be expressed by

$$f_\Psi(X)=\begin{cases}(1-q_1)(1-q_2) & \text{if } X=\varnothing \\ q_1(1-q_2)\cdot f_1(\mathbf{x})+(1-q_1)q_2\cdot f_2(\mathbf{x}) & \text{if } X=\{\mathbf{x}\} \\ q_1q_2\cdot\left[f_1(\mathbf{x}_1)f_2(\mathbf{x}_2)+f_1(\mathbf{x}_2)f_2(\mathbf{x}_1)\right] & \text{if } X=\{\mathbf{x}_1,\mathbf{x}_2\} \\ 0 & \text{if } |X|>2\end{cases}$$

Then the combined PHD can be expressed by

$$D_\Psi(\mathbf{x})=q_1\cdot f_1(\mathbf{x})+q_2\cdot f_2(\mathbf{x}).$$

Assume that these two 2-D PDFs are expressed by

$$f_1(\mathbf{x})=w_1\frac{1}{\sqrt{2\pi|\Sigma_1|}}e^{-(\mathbf{x}-\mu_1)^T\Sigma_1^{-1}(\mathbf{x}-\mu_1)}+w_2\frac{1}{\sqrt{2\pi|\Sigma_2|}}e^{-(\mathbf{x}-\mu_2)^T\Sigma_2^{-1}(\mathbf{x}-\mu_2)}$$

$$f_2(\mathbf{x})=\frac{1}{\sqrt{2\pi|\Sigma_3|}}e^{-(\mathbf{x}-\mu_3)^T\Sigma_3^{-1}(\mathbf{x}-\mu_3)}$$

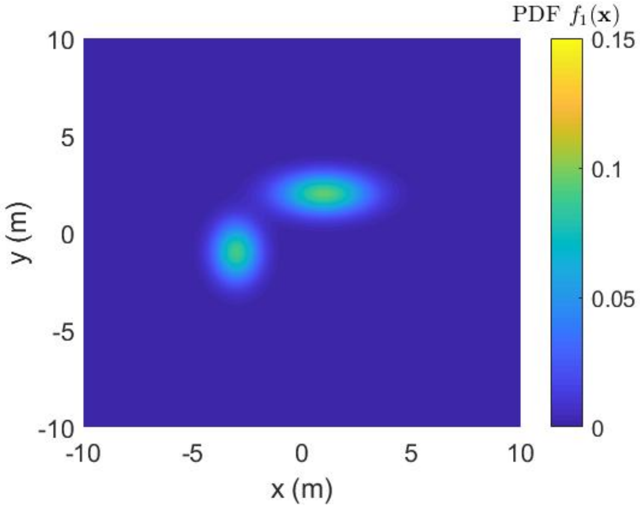
$$q_1=0.9 \qquad q_2=0.5$$

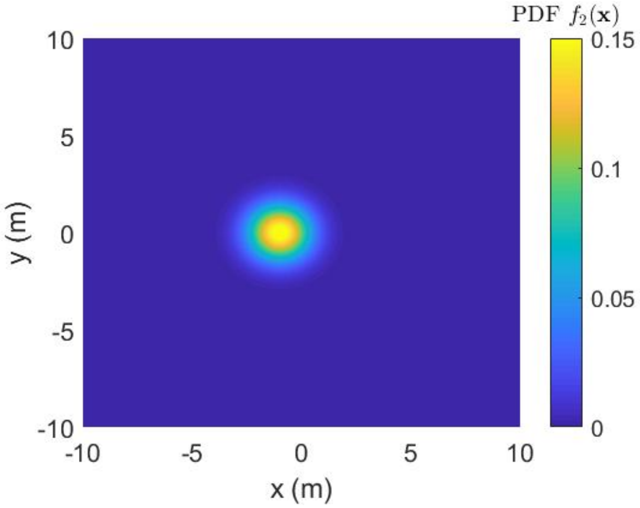
$$\mu_1=[1,2]^T \quad \mu_2=[-3,-1]^T \quad \mu_3=[-1,0]^T$$

$$\Sigma_1=\begin{bmatrix}2&0\\0&0.5\end{bmatrix} \quad \Sigma_2=\begin{bmatrix}0.5&0\\0&1\end{bmatrix} \quad \Sigma_3=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

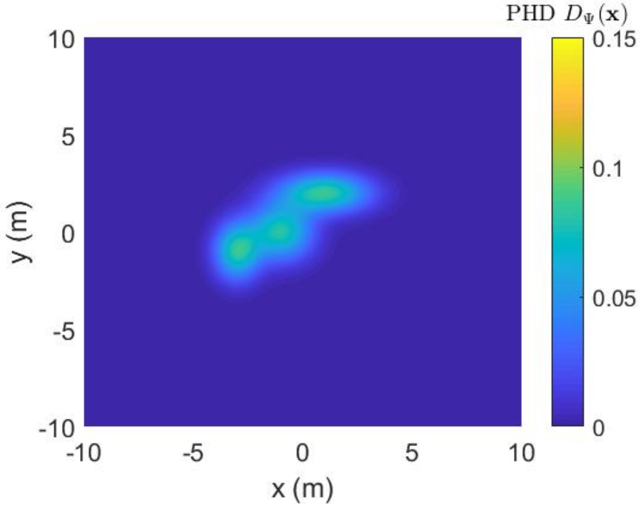
Example: Multi-Bernoulli RFS

- The two 2-D PDFs are plotted below:





- The combined PHD is



Example: Poisson RFS

- Assume the locations of targets in a work space $\mathcal{W} = [-10m, 10m] \times [-10m, 10m]$ are described by a Poisson RFS Ψ with a initial PHD $D_0(\mathbf{x})$. The expected number of target in the work space is equal to 5, such as

$$N_0 = \int_{\mathcal{W}} D_0(\mathbf{x}) d\mathbf{x} = 5$$

where the target location $\mathbf{x} = [x, y]^T \in \mathcal{W} \subset \mathbb{R}$ and the initial PHD $D_0(\mathbf{x})$ is defined by

$$D_0(x, y) = \left[\frac{\sin(\sqrt{x^2 + y^2})}{\sin(\sqrt{x^2 + y^2 + 0.1})} + 0.5 \right] \cdot a$$

where a is a normalization term.

The PDFs of targets are identical and can be expressed by

$$\wp(x, y) = \frac{D_0(x, y)}{N_0} = \frac{D_0(x, y)}{5}$$

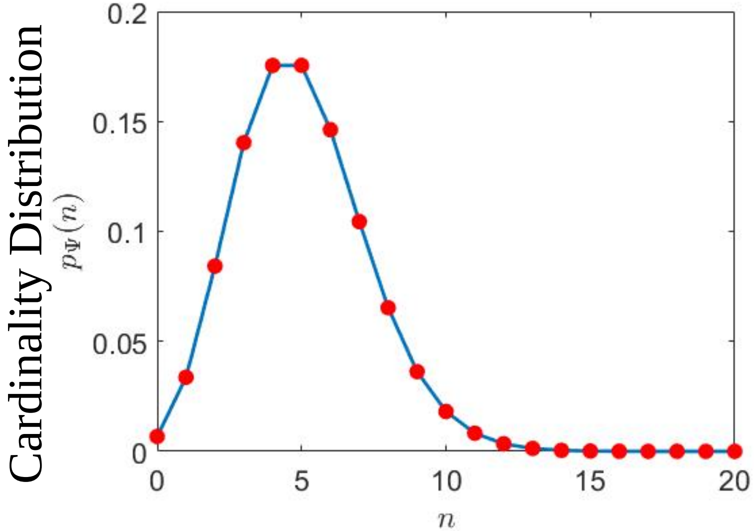
and the MPDF of the targets in the work space is expressed by

$$f_\Psi(X) = e^{-N_0} \cdot N_0^X = e^{-5} \cdot (5\wp)^{(X)}$$

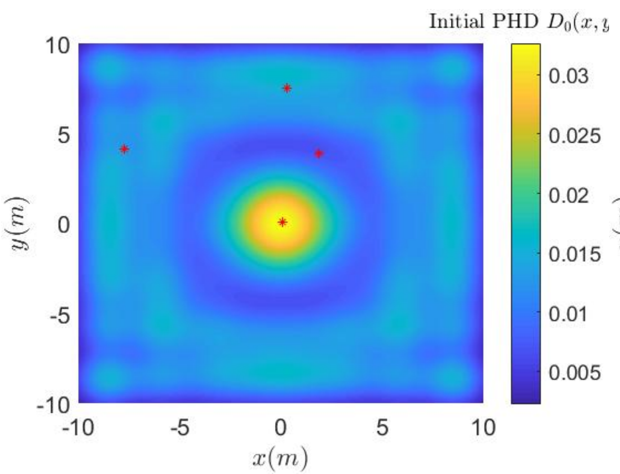
Example: Poisson RFS

Three samples of RFS Ψ are plotted below:

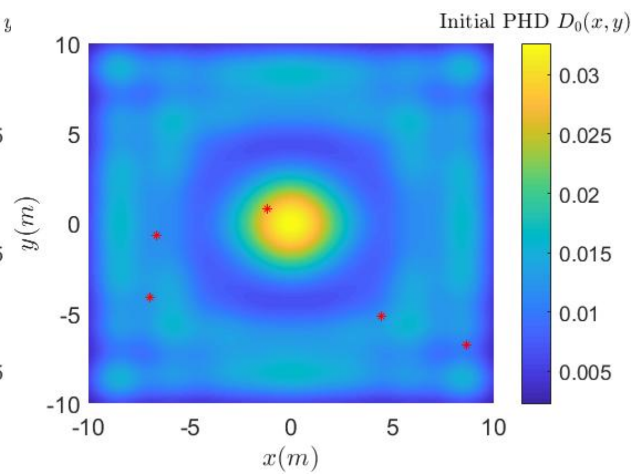
- The cardinality distribution is a Poisson distribution
$$p_\Psi(n) = \frac{\lambda^n e^{-N_0}}{n!} = \frac{\lambda^n e^{-5}}{n!}$$
- The numbers of targets in the work space are 4, 5, and 7, respectively.
- The Poisson RFS is special case of the *i.i.d.c.* RFS, where the cardinality distribution is a Poisson distribution.



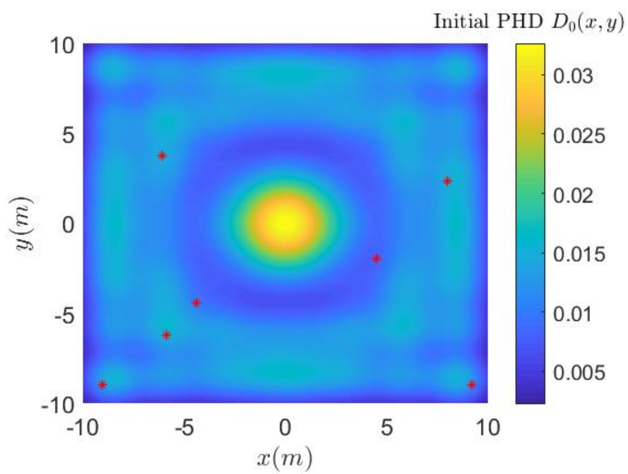
Three different realizations of the same RFS



(a) 4 Agents



(b) 5 agents



(c) 7 agents

DOC Theory and Analysis via FISST

By introducing the FISST framework in DOC, a RFS Ψ can represent the macroscopic state and can be used to describe the network, where the finite set of the states of all *n* agents in the ROI, $X = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, is treated as a realization of the RFS Ψ.

- The microscopic states of these agents can treated as continuous random variables denoted by $\mathbf{X}_i,\ i = 1, \ldots n$.
- The number of agents, *n*, is also a realization of the discrete random variable *N*.

Assume that the macroscopic state is a *identical,independently distributed cluster* (i.i.d.c) RFS, where PDFs of the agents are assume to be the same, denoted by $\wp(\mathbf{x})$. Then, the MPDF of the RFS, Ψ, can be expressed by

$$f_\Psi(X) = |X|! \cdot p_\Psi(|X|) \cdot \wp^X$$

The expected number of the agents denoted by N_Ψ can be expressed by

$$N_\Psi = \sum_{n \geq 0} n \cdot p_\Psi(n)$$

and the PHD of the RFS can be expressed by

$$D_\Psi(\mathbf{x}) = N_\Psi \cdot \wp(\mathbf{x})$$

DOC Results: Number of Assets

In the DOC method, the number of the agents, N_{agent}, is a constant. The cardinality distribution can be expressed by

$$p_{\Psi}(n) = \begin{cases} 1 & \text{if } n = N_{agent} \\ 0 & \text{if } n \neq N_{agent} \end{cases}$$

Then, the MPDF in the DOC problem can be expressed by

$$f_{\Psi}(X) = \begin{cases} N_{agent} \cdot \wp^{X} & \text{if } |X| = N_{agent} \\ 0 & \text{if } |X| \neq N_{agent} \end{cases}$$

and the PHD of the agents can be expressed by

$$D_{\Psi}(\mathbf{x}) = N_{agent} \cdot \wp(\mathbf{x})$$

Information Value with Limited Communications

Information Value Analysis

The Expected Entropy Reduction (EER) of a measurement set Z at location $\mathbf{x}$ at time k can be defined as the following:

$$\Delta H(Z_k(\mathbf{x})) = H(Y \mid \mathcal{M}_{k-1}, \lambda) - \mathbb{E}_Z[H(Y \mid Z_k(\mathbf{x}), \mathcal{M}_{k-1}, \lambda)]$$

The Total Expected Entropy Reduction (Total EER) can be defined for the system of robots as:

$$\Delta H_{tot}(k) := \int_{\mathcal{X}} \Delta H(Z_k(\mathbf{x}))d\mathbf{x}$$

The Average Total EER can be defined for the system of robots as:

$$\overline{\Delta H_{tot}}(k) := \frac{1}{N}\sum_{j=1}^{N} \Delta H_{tot}(j)$$

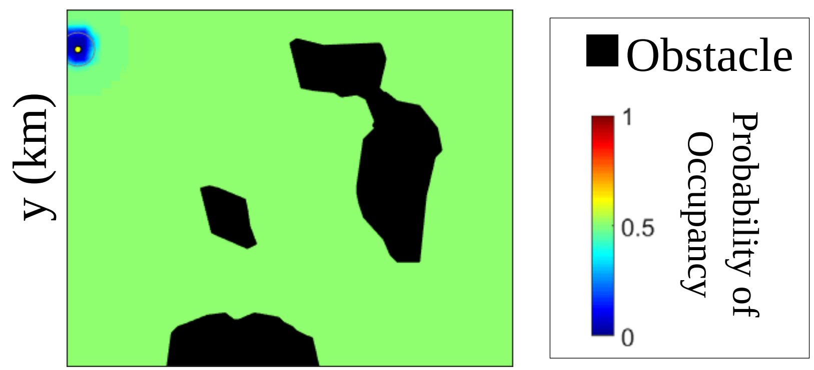
Total number of agents N

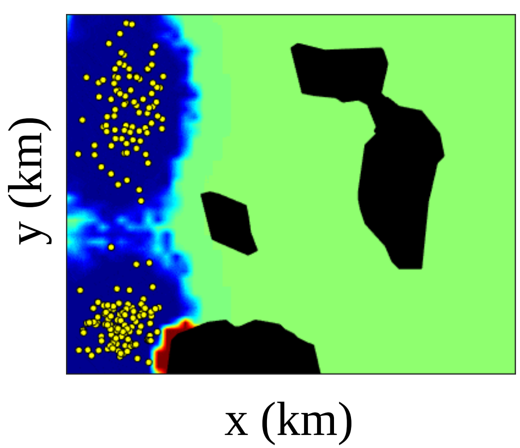
Z_k : measurement at location $\mathbf{x}$ at time k
Y : binary categorical random variable
$\mathcal{M}_k$: set of all previous measurements up to time k
λ : vector of environmental conditions

Influence of Limited Communications

- Consider the problem of searching and information about targets and environments (DCLT)
- A network of sensors can provide better performance than a single agent provided they can communicate

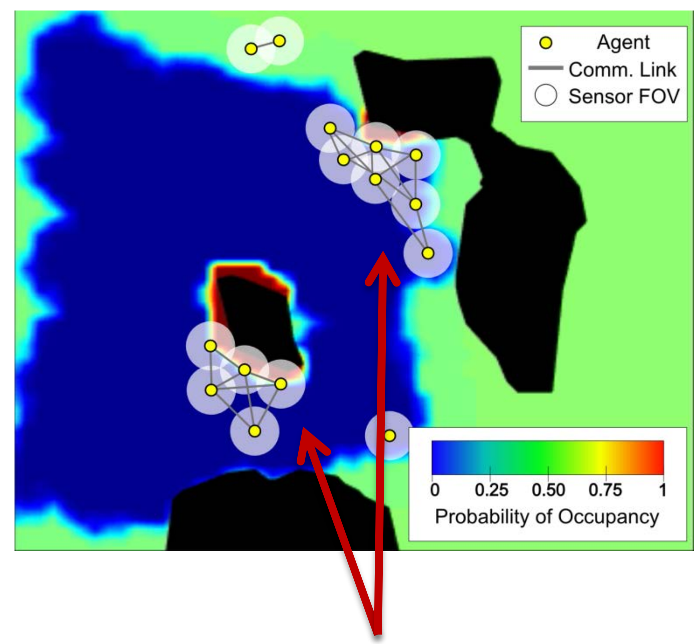
Resulting Hilbert Map





Full Communications

Partial Communications

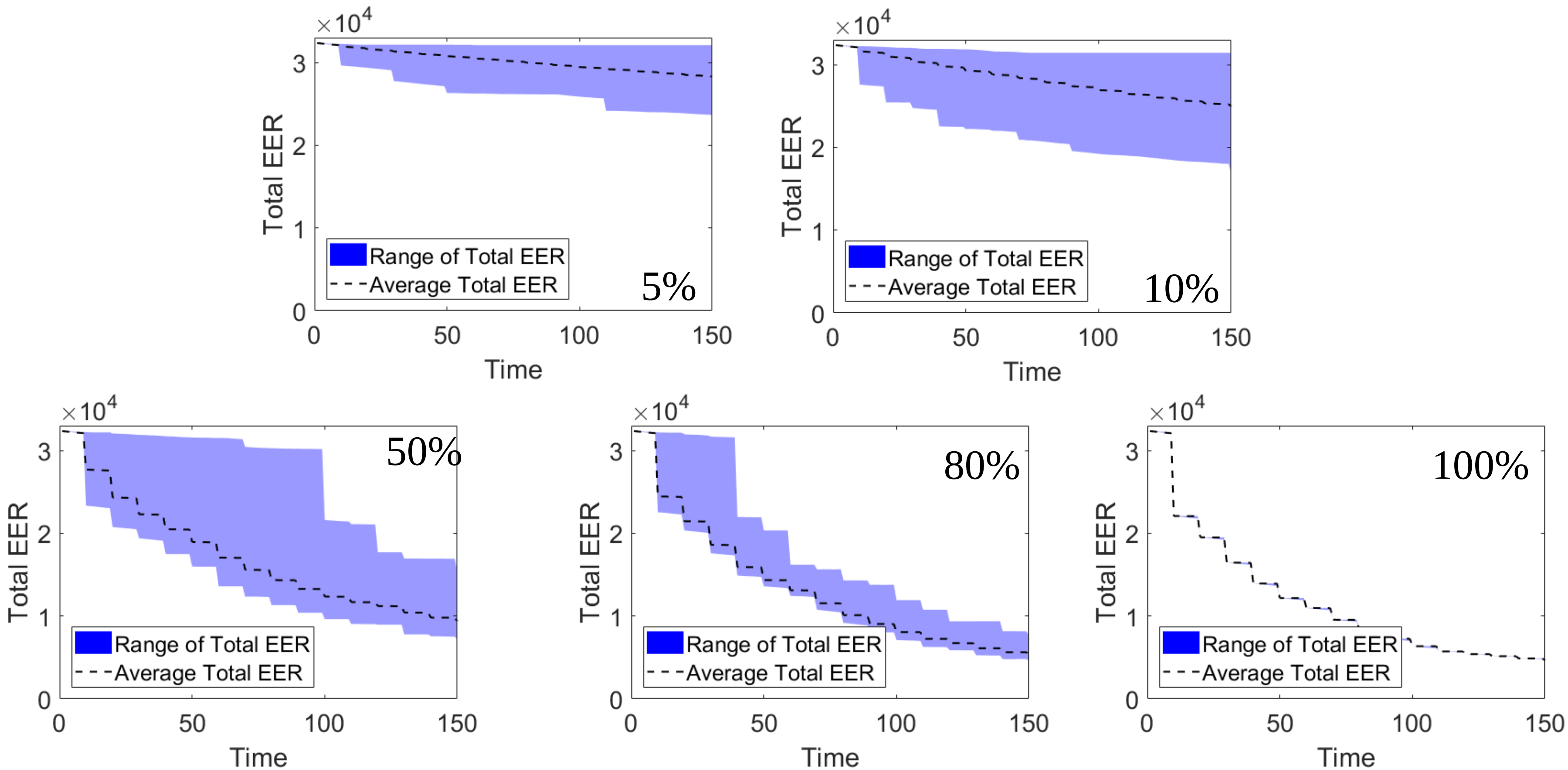


Disconnected communication graphs

Information Value as a Function of Network Connectivity

Numerical Results:

Total and average expected entropy reduction (EER) for a network of $N = 200$ vehicles with varying communication abilities (% of agents).



Preliminary Results

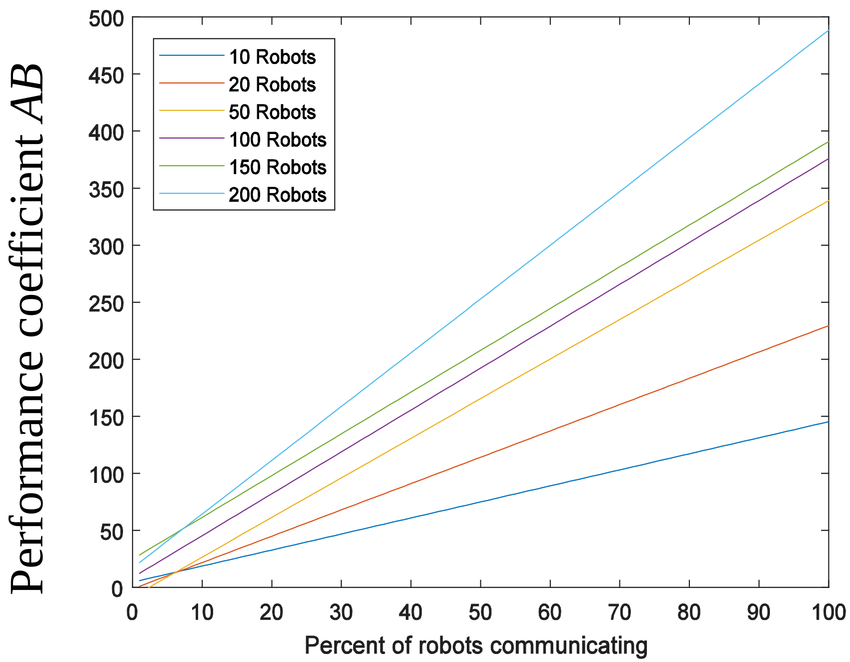
The Average Total EER can be fit to a one-component exponential function

$$\overline{\Delta H_{tot}}(k) \approx Ae^{-Bk}$$

By taking its derivative, the Average Total EER rate of the system can be obtained

$$\frac{d}{dk}(\overline{\Delta H_{tot}}(k)) = -ABe^{-Bk}$$

Performance Coefficient



Number of robots	R-squared value
10	.9635
20	.9876
50	.9903
100	.9923
150	.9507
200	.9913

Multi-scale Environmental Adaptation

Multi-Scale Adaptive DOC

- *Adaptive* distributed optimal control:
  - Agents make decisions based on *in situ* conditions and *environment* information
  - Value function V, defined in terms of discrete agent distribution $\wp_k$, Hilbert map h_k at time k and control law $C(\wp_k)$:

$$V \equiv \varphi[\wp_{k_f}] + \sum_{t_k}^{t_{f-1}} \int_W \mathscr{L}[\wp_k, h_k, C(\wp_k)] d\mathbf{x}_i = V[\wp_k, h_k, C(\cdot)]$$

- Control law C_l and Value function V_l are iteratively improved online, where l is the iteration

Policy-Improvement Routine:

$$C_{l+1}(\wp_k) = \arg\min_{\mathbf{u}_k} \left\{ \int_W \mathscr{L}(\wp_k, h_k, \mathbf{u}_k) d\mathbf{x}_i + V[\wp_{k+1,} h_k, C_l(\cdot)] \right\}$$

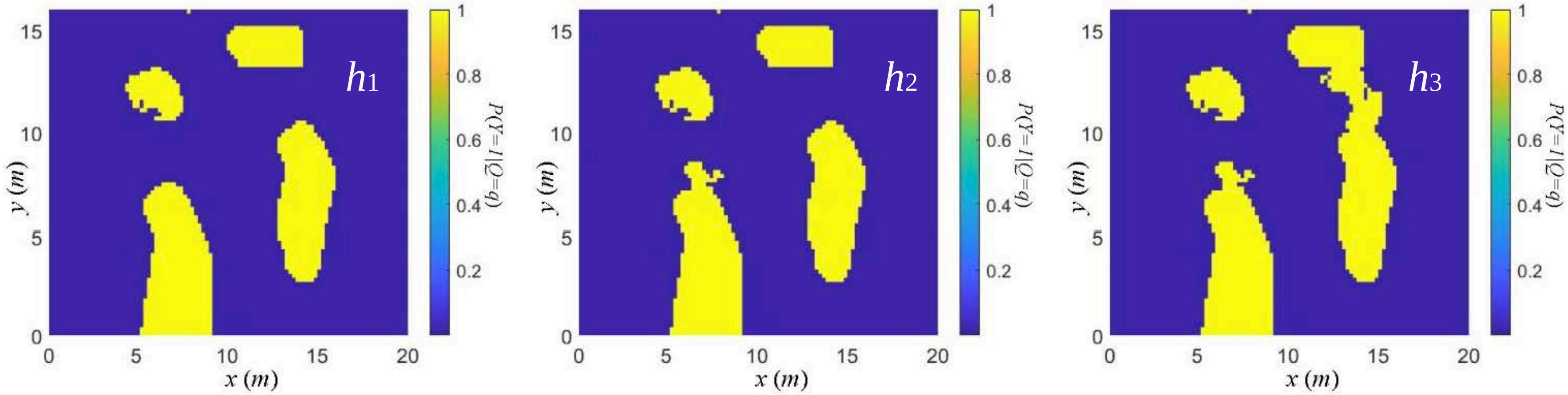
Value-Determination Operation:

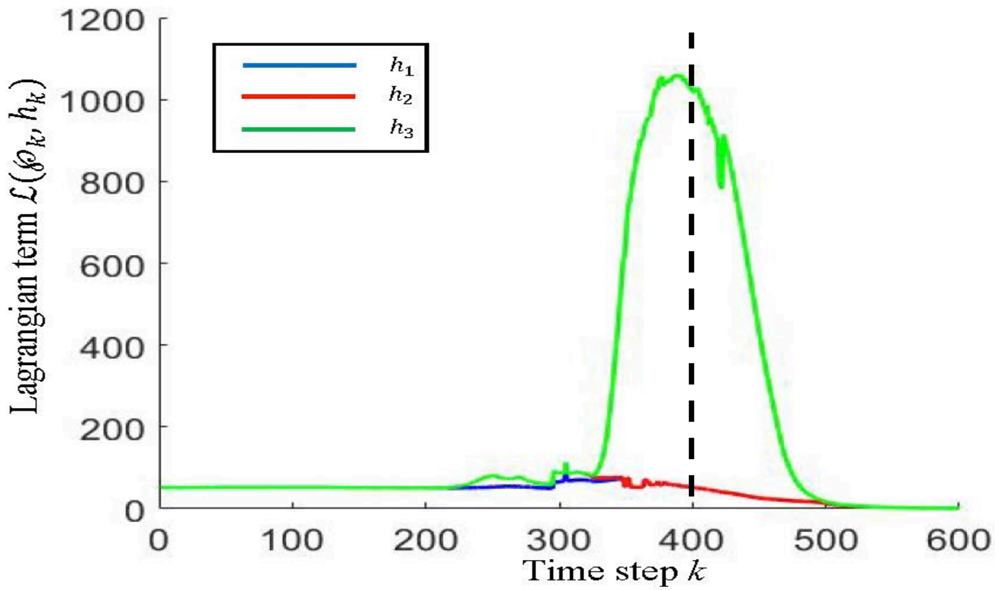
$$V_{l+1}[\wp_k, C(\cdot)] = \int_W \mathscr{L}(\wp_k, h_k, \mathbf{u}_k) d\mathbf{x}_i + V_l[\wp_{k+1}, h_k, C(\cdot)]$$

Optimal value function V^*:

$$V^* = \min_{\mathbf{u}_k} \left\{ \int_X \mathscr{L}(\wp^*{}_k, h_k, \mathbf{u}_k) d\mathbf{x}_i + V^*[\wp^*{}_{k+1}, h_k, C^*(\cdot)] \right\}$$

Simulation Results





Summary and Conclusions

Multi-scale Adaptive Optimal Control:

- ✓ Recurrent relations for policy improvement and value iteration
- ✓ Decentralized Hilbert mapping for information fusion
- ✓ Communication protocols for efficient map-information spreading
- ✓ FISST analysis of DOC approach
- ✓ Performance analysis: connectivity influence on information value
- ✓ Environmental adaptation: adaptive value function

Future Work:

- ➢ Stability analysis in the presence of delayed information propagation
- ➢ Environmental adaptation: optimize agent density over time
- ➢ Robustness and performance analysis

Acknowledgements

- **Collaborators:**

- **Pingping Zhu**, Ph.D.
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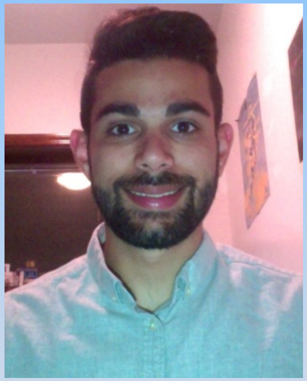
- **Julian Morelli**,
  LISC, MAE, Cornell



- **Collaborators:**

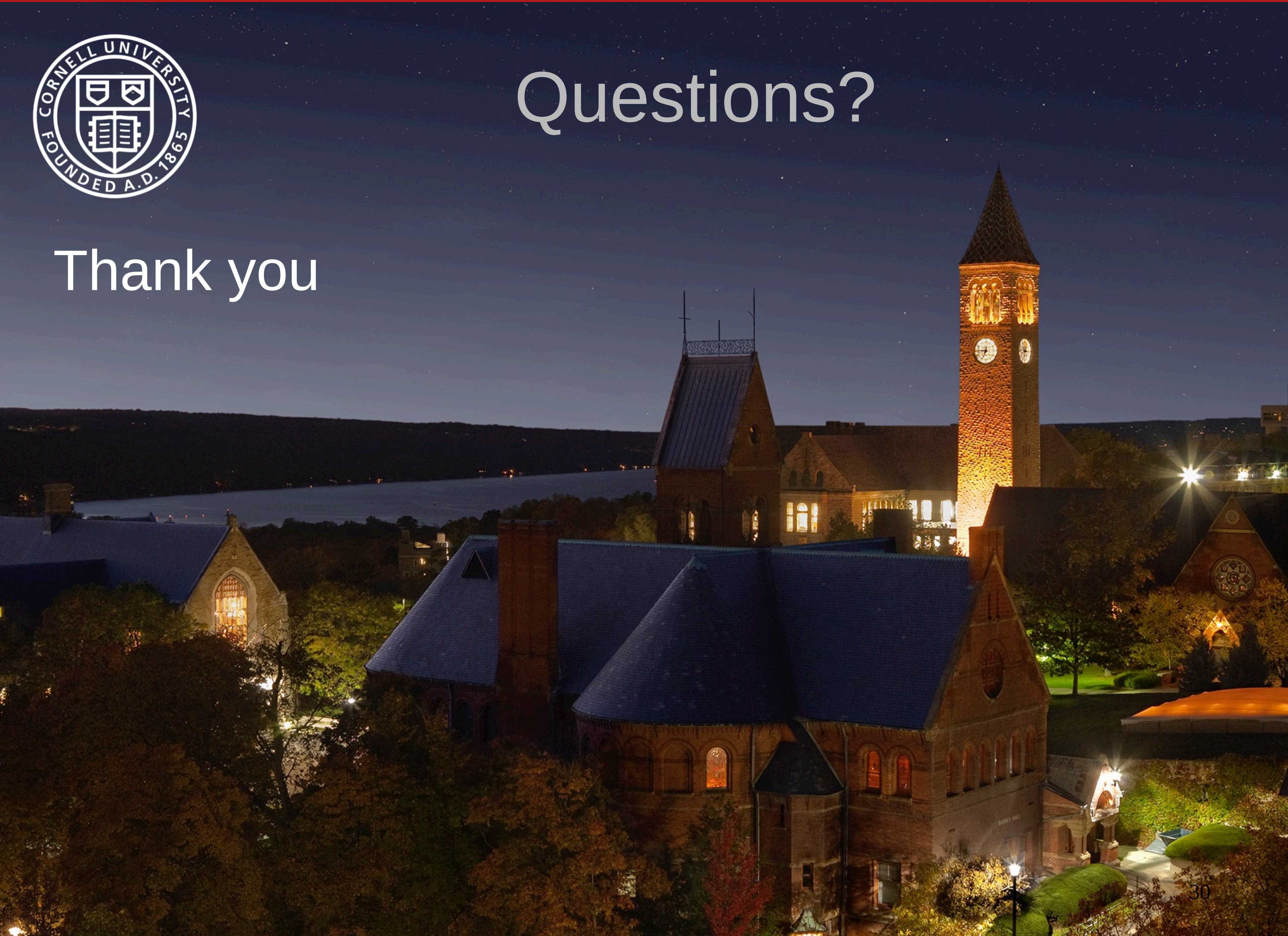
This work was funded by ONR Code 321

Important Events and Publications

- Invited Talk: ***Workshop on Informative Path Planning and Adaptive Sampling***; 2018 IEEE International Conference on Robotics and Automation (ICRA 2018) Brisbane, Australia; May 21, 2018
- Invited Talk: ***Workshop on Imaging Beyond the Visible Spectrum***; SIAM Imaging Science Symposium, IS18; Bologna, Italy, June 5th 2018
- G. Foderaro, S. Ferrari, T. A. Wettergren, "Distributed Optimal Control of Sensor Networks for Dynamic Track Coverage," *IEEE Transactions on Control of Network Systems*, Vol. 5, No. 1, 2018.
- K. Rudd, G. Foderaro, and S. Ferrari, "A Generalized Reduced Gradient Method for the Optimal Control of Very Large Scale Robotic (VLSR) Systems," *IEEE Transactions on Robotics*, Vol. 33, No. 5, pp. 1226-1232, 2017.
- S. Ferrari, G. Foderaro, P. Zhu, and T.Wettergren "Distributed Optimal Control of Multiscale Dynamical Systems: A Tutorial", *Control Systems Magazine*, Vol. 36, No. 2, pp. 102-116, 2016.
- G. Foderaro, S. Ferrari, T. A. Wettergren, "Distributed Optimal Control for Multi-agent Trajectory Optimization," *Automatica*, Vol. 50, No. 1, pp. 149-154, 2014.

Questions?

Thank you



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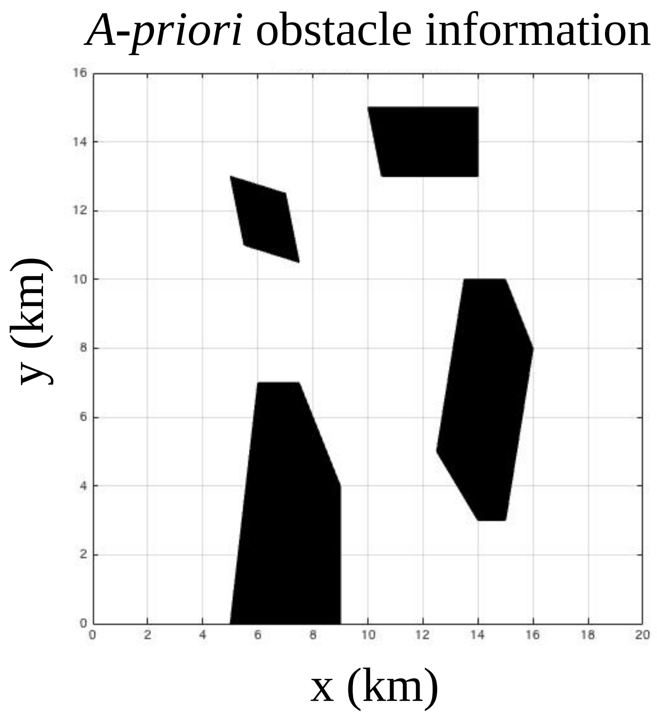
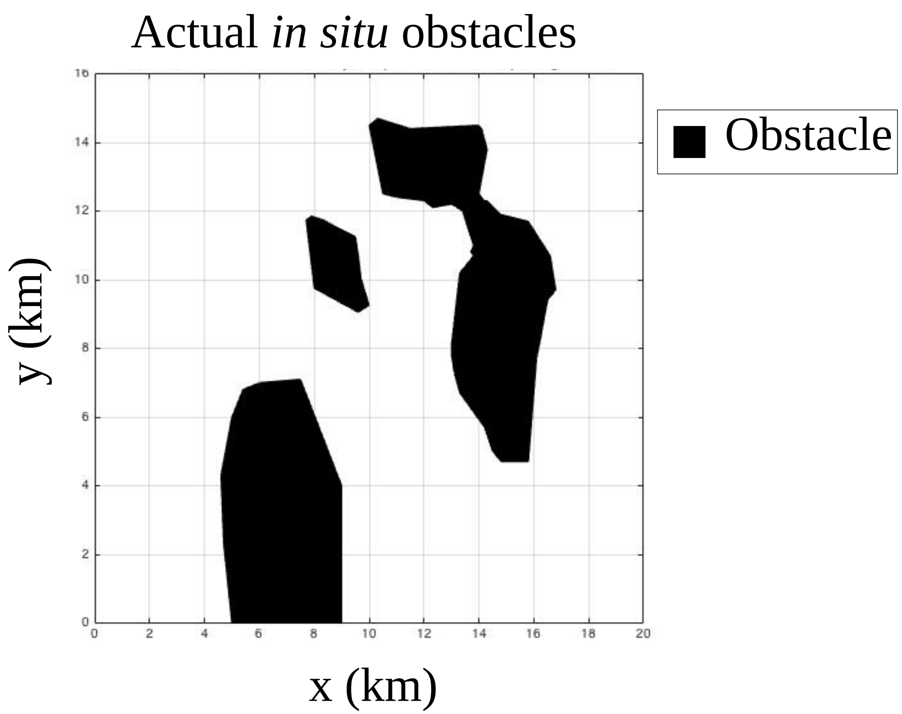
Backup Slides

Motivation: Multi-scale Adaptive Control

- Environmental and operating conditions *in situ* may drastically differ those used *a priori*
  - – Off-line DOC solutions no longer optimal
  - – Agents must react to local information, including new tactical constraints
  - – Network-level controller can dispatch agents to localize gradient intensification while providing energy management, volume coverage, and robustness to component failure





Motivation: Oceanographic Conditions

- Full, nonlinear dynamics $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)$
- Robot state: $\mathbf{x} = [x, y, z, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, \dot{\theta}, \dot{\psi}]^T$

  Inertial position (x, y, z) | Pitch and yaw Euler angles (θ, ψ)

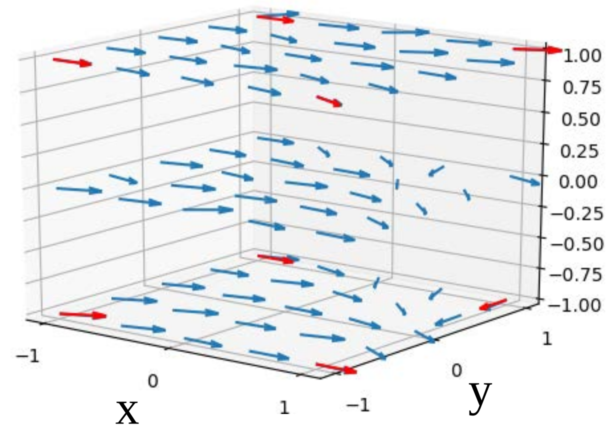
- Robot Control: $\mathbf{u} = [\delta_{rpm}, \delta_r, \delta_s]$

- UUV kinematics:

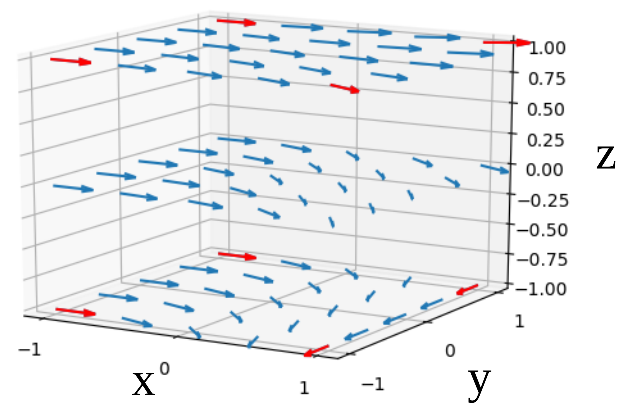
$$\dot{x} = \|\mathbf{v}_B\|(\cos\theta\cos\psi) + v_{cx}$$
$$\dot{y} = \|\mathbf{v}_B\|(\cos\theta\sin\psi) + v_{cy}$$
$$\dot{z} = \|\mathbf{v}_B\|(-\sin\theta)$$
$$\dot{\theta} = g_\theta \delta_r$$
$$\dot{\psi} = g_\psi \delta_s$$

$\mathbf{v}_B$: measured velocity
v_{cx}, v_{cy}: measured ocean and y velocities
g_θ, g_ψ : control gains

A-priori current estimates:

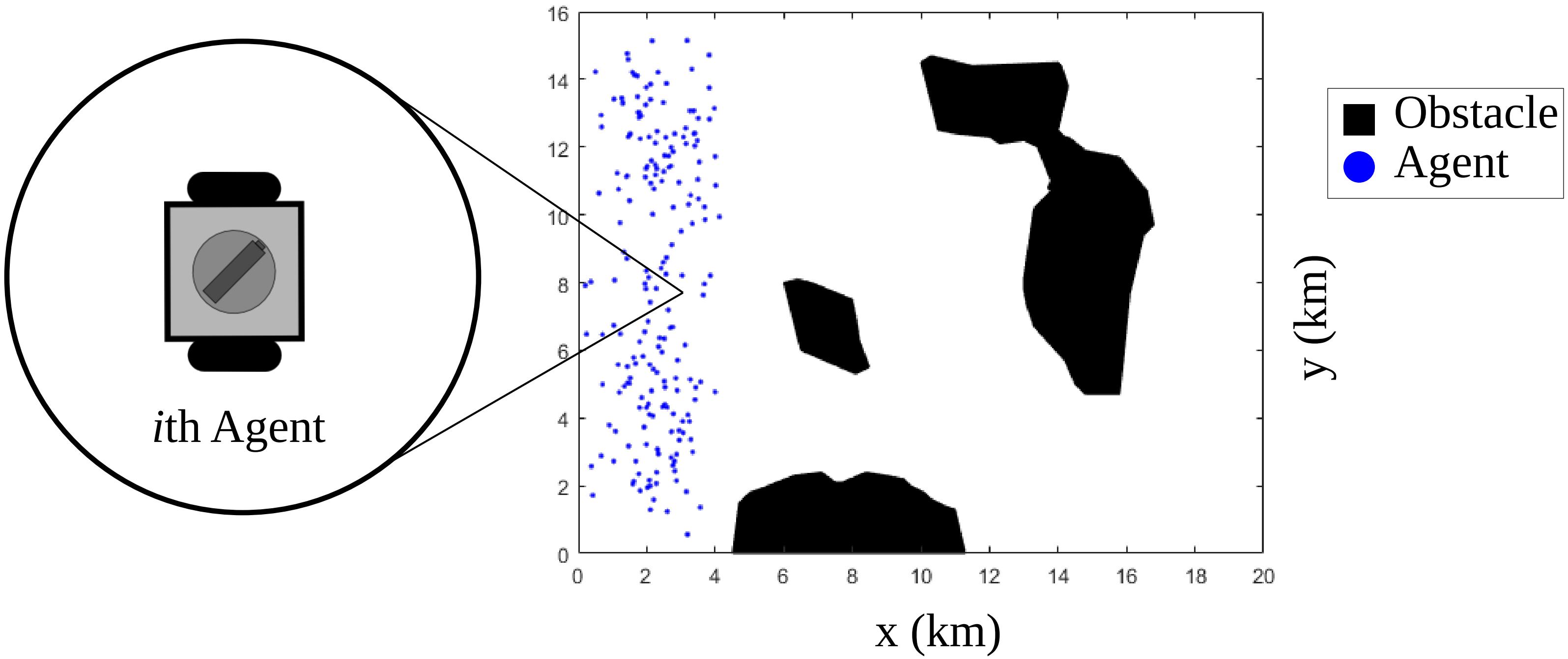


Actual currents:



Problem Formulation

Mission Goal: Collectively *explore* and *map* obstacles and currents in a region of interest *W* while obtaining decentralized sensor measurements, avoiding obstacles, and communicating with other agents and a central station.



- Region of Interest $W \subset R^2$:
- Fixed, unknown, rigid obstacles, B_i , $i=1,\ldots,r$
- N agents

On-board Sensor Measurements

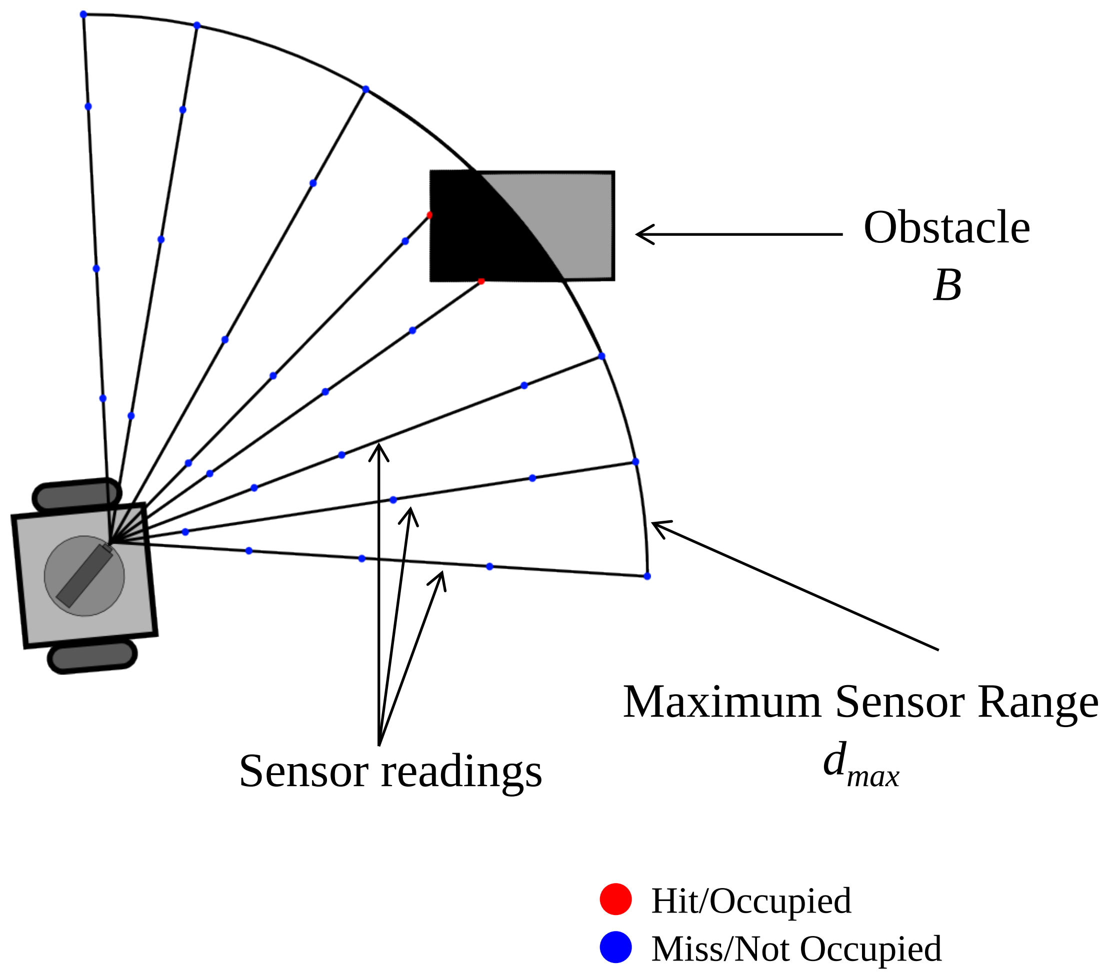
- Sensor can infer classification $\hat{Y}$ within sensor range and construct $D_i = \{\mathbf{x}_j, y_j\}^m$
- Noisy sensor measurements:

$$\mathbf{x}_M = \mathbf{x}_i + D\hat{\mathbf{e}}_r$$

$$\hat{Z} = [\hat{\mathbf{x}}_M, \hat{Y}]$$

- Y is a random and binary classification variable:

$$Y(\mathbf{x}) = \begin{cases} 1, \text{if } \mathbf{x} \in B \\ 0, \text{if } \mathbf{x} \notin B \end{cases}$$



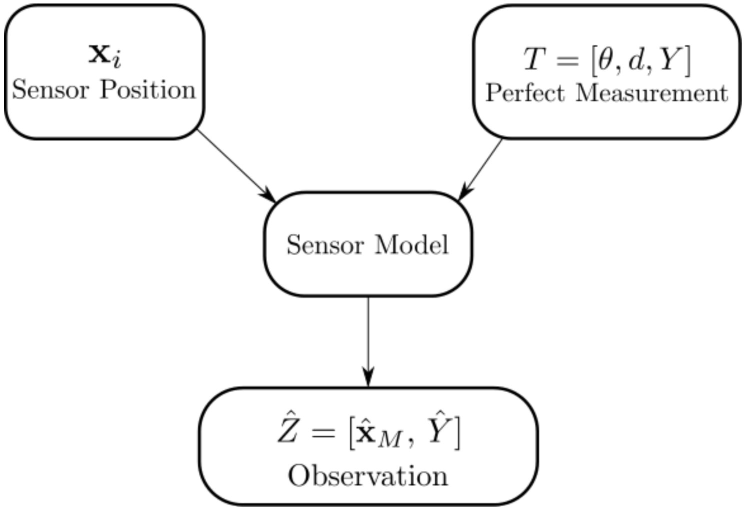
D = $d + v$: Distance measurement
$\Theta = \theta + \upsilon$: Angle measurement
$v \sim N(0,\sigma_d)$: Sample from normal distribution
$\upsilon \sim N(0, \sigma_\theta)$: Sample from normal distribution
$\hat{\mathbf{e}}_r = \hat{x}\cos\Theta + \hat{y}\sin\Theta$: radial unit vector
$\hat{x}, \hat{y}$: unit vectors of basis in F_A

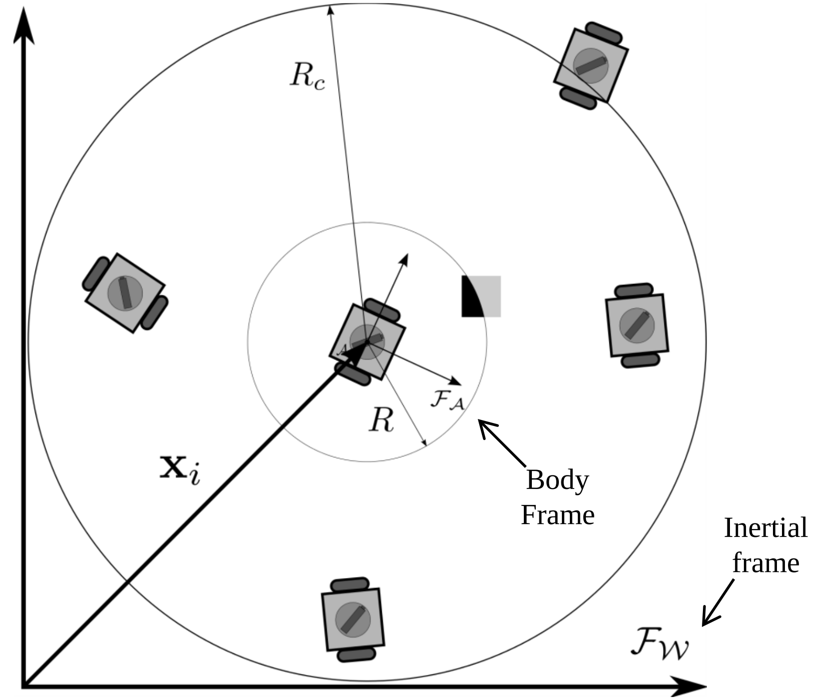
On-board Sensing and Communications

- **Sensing goal:** maximize information gain of future measurements to minimize uncertainty
- **Mission constraints:**
  Bounded sensor FOV range, R
  Bounded communication range, R_c
- **Multiobjective Cost Function:**

$$J = \int_W \hat{\varphi}(Y;Z\,|\,M,\lambda)\,d\mathbf{x} + \int_{T_0}^{T_f} \left\{ U[\mathbf{x}_i(t)] + \mathbf{u}_i(t)^T \mathbf{R}\mathbf{u}_i(t) \right\} dt$$

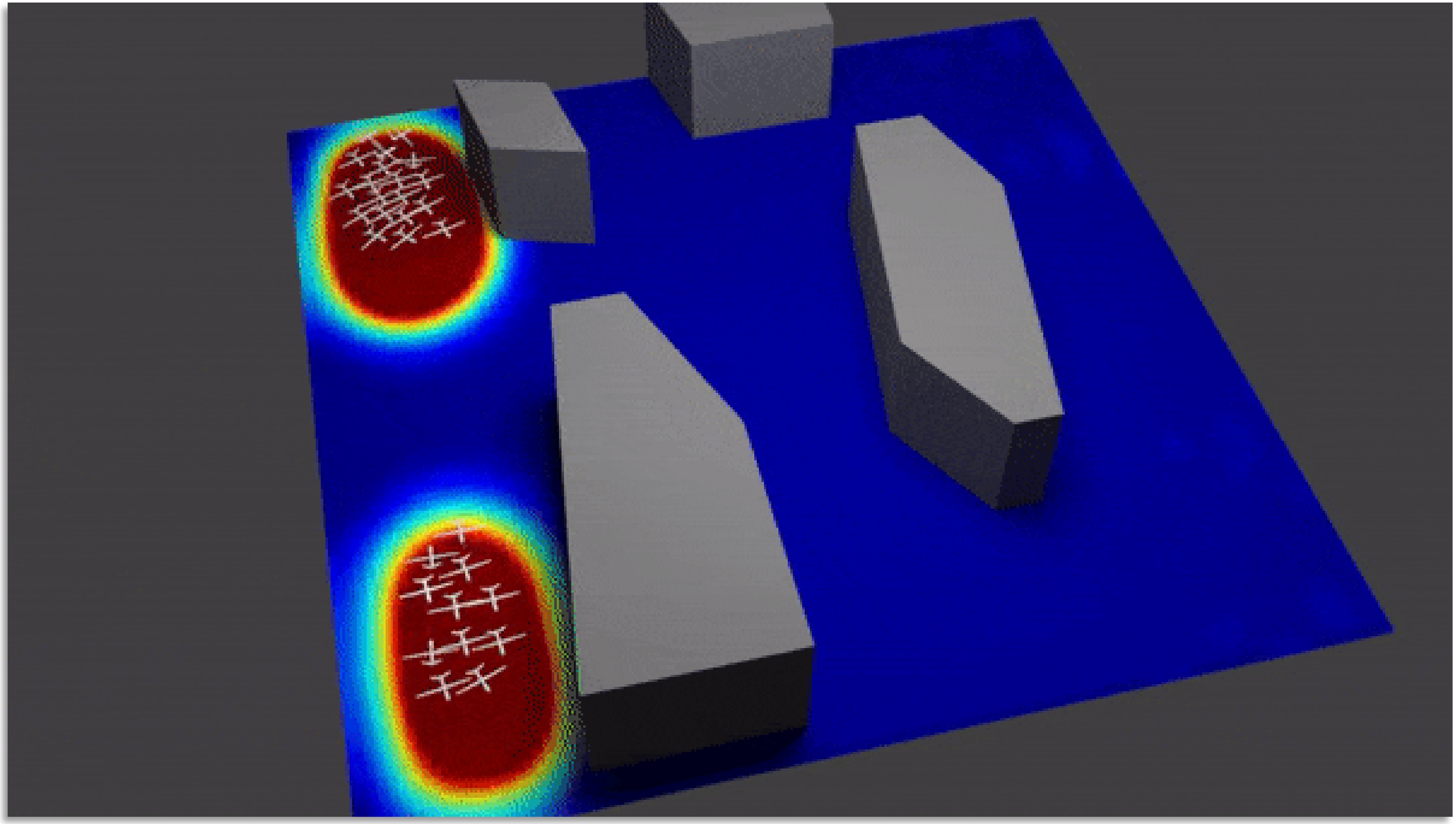
Bayesian measurement model: $p(Z\,|\,Y,\lambda)$





- $\hat{\varphi}(\cdot)$: Information gain
- Y, Z: hidden discrete random variables
- M: set of all prior measurements
- λ: environmental condition parameters
- $U(\mathbf{x}_i)$: obstacle repulsion potential
- R: control weight matrix

An Illustrative DOC Example



Value Function Approximation

Consider the training data set $\mathcal{D}^0 = \{(\wp_i^0, h^0, \ell_i^0)\}_{i=1}^N$ and the testing data set $\mathcal{D} = \{(\wp_i, h, \ell_i)\}_{n=1}^N$ The training and the testing data sets are all generated by the same control law $C(\cdot)$. The goal of value function approximation is to approximate an operator $\widehat{\mathcal{V}}^{\mathcal{C}}(\cdot,\cdot)$ from the training data set $\mathcal{D}^0$ and predict the evaluation of the value function in the testing data set $\mathcal{D}$

First, consider that the Hilbert map function h^0 is fixed. Then, the approximation of the value function can be expressed by

$$\widehat{\mathcal{V}}_{h^0}^{\mathcal{C}}(\wp) = \widehat{\mathcal{V}}^{\mathcal{C}}(\wp, h^0)$$

To learn the operator, a new kernel least squares temporal difference (KLSTD) algorithm is proposed based on a functional kernel, which is defined as follows:

$$k(\wp_i, \wp_j) = \exp\left(\frac{\lVert \wp_i - \wp_j \rVert_{\mathcal{P}}^2}{\sigma_{\wp^2}}\right) = \exp\left(\frac{\langle \wp_i, \wp_i \rangle_{\mathcal{P}} - 2\langle \wp_i - \wp_j \rangle_{\mathcal{P}} + \langle \wp_j, \wp_j \rangle_{\mathcal{P}}}{\sigma_{\wp^2}}\right)$$

where the inner product is defined by

$$\langle \wp_i, \wp_j \rangle_{\mathcal{P}} = \int_{\mathcal{W}} \wp_i(\mathbf{x}) \wp_j(\mathbf{x}) d\mathbf{x}$$

Value Function Approximation

Let $\phi(\wp) = k(\wp, \cdot)$ denote the kernel feature map and $\Phi = [k(\wp_1, \cdot), \dots, k(\wp_N, \cdot)]$ denote the feature matrix. Then, according to the KLSTD algorithm, the value function $\mathcal{V}(\cdot, h^0)$ is approximated by

$$\widehat{\mathcal{V}}_{h^0}^{\mathcal{C}}(\wp) = \mathbf{k}(\wp)^T \mathbf{H}^T \left(\mathbf{HKH}^T\right)^{-1} \mathbf{KL}$$

Here,

$$\mathbf{K} = \begin{bmatrix} k(\wp_1, \wp_1) & \dots & k(\wp_1, \wp_N) \\ \vdots & \ddots & \vdots \\ k(\wp_N, \wp_1) & \dots & k(\wp_N, \wp_N) \end{bmatrix} = \Phi^T \Phi \qquad \mathbf{H} = \begin{bmatrix} 1 & -\gamma & 0 & \dots & 0 \\ 0 & 1 & -\gamma & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \dots & 1 & -\gamma \\ 0 & 0 & & 0 & 1 \end{bmatrix}$$

and

$$\mathbf{L} = \left[\mathcal{L}[\wp_1, h_0, \mathcal{C}(\cdot,\cdot)], \dots, \mathcal{L}[\wp_N, h_0, \mathcal{C}(\cdot,\cdot)]\right]^T$$

Therefore, the approximated value function can be expressed by

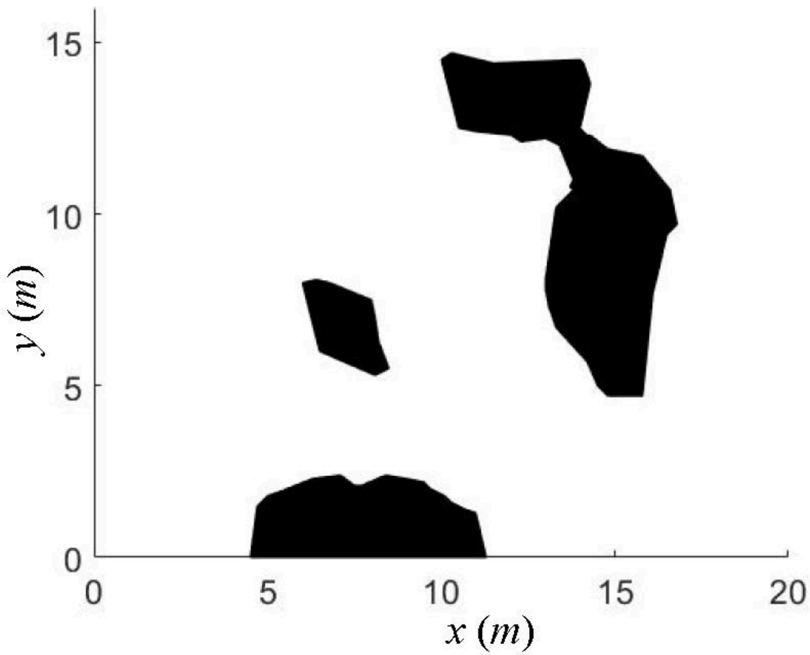
$$\widehat{\mathcal{V}}_{h^0}^{\mathcal{C}}(\wp) = \phi(\wp)^T \Phi \mathbf{H}^T \left(\mathbf{HKH}^T\right)^{-1} \mathbf{KL}(\Phi, h^0) = \phi(\wp)^T \Phi \mathbf{F}(\Phi) \mathbf{L}(\Phi, h^0)$$

where $\mathbf{F}(\Phi) = \mathbf{H}^T \left(\mathbf{HKH}^T\right)^{-1} \mathbf{K}$

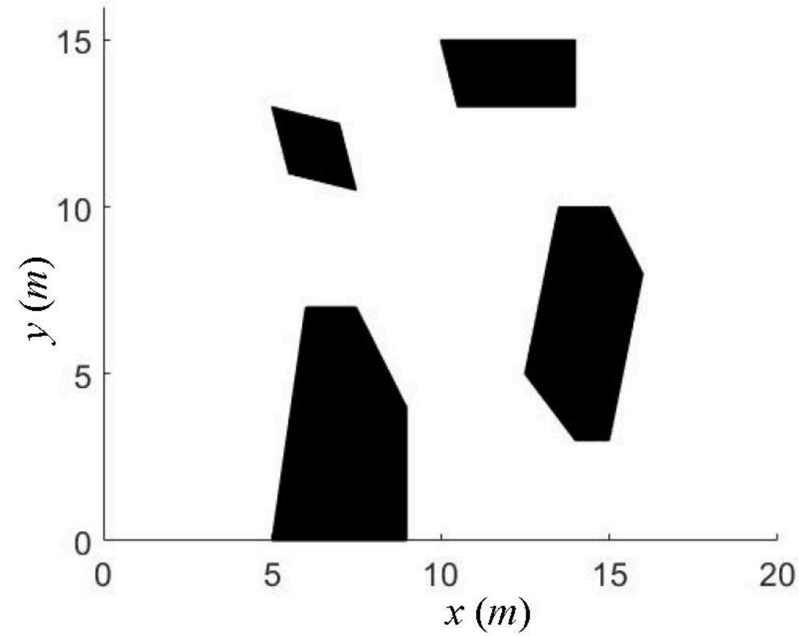
Furthermore, if the Hilbert map h^0 is varying, then the approximated value function with two function arguments can be expressed by

$$\widehat{\mathcal{V}}^{\mathcal{C}}(\wp, h) = \phi(\wp)^T \Phi \mathbf{F}(\Phi) \mathbf{L}(\Phi, h)$$

Simulation Results



Actual environment



Idea environment

- The environment is approximated by Hilbert map *h*.
- The Hilbert map is updated by the new measurements obtained by a group of agents

- The idea environment is assumed to be known in advance.
- The trajectory of optimal agent distributions, $\wp(k),\ k = 1, \ldots, 600$, is obtained by the DOC method.

Simulation Results